Exercises

Question 9.1:
A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

Answer 9.1:
Size of the candle, h = 2.5 cm
Image size = h'
Object distance, u = −27 cm
Radius of curvature of the concave mirror, R = −36 cm
Focal length of the concave mirror, $f = \frac{R}{2} = -18\ cm$
Image distance = v
The image distance can be obtained using the mirror formula: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-18} - \frac{1}{-27} = -\frac{1}{54}$$
$$\Rightarrow v = -54\ cm$$

Therefore, the screen should be placed 54 cm away from the mirror to obtain a sharp image.
The magnification of the image is given as:

$$m = \frac{h'}{h} = -\frac{v}{u}$$
$$\therefore h' = -\frac{v}{u} \times h$$
$$= -\left(\frac{-54}{-27}\right) \times 2.5 = -5 \text{ cm}$$

The height of the candle's image is 5 cm. The negative sign indicates that the image is inverted and virtual.
If the candle is moved closer to the mirror, then the screen will have to be moved away from the mirror in order to obtain the image.

Question 9.2:
A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.

Answer 9.2:
Height of the needle, h1 = 4.5 cm
Object distance, u = −12 cm
Focal length of the convex mirror, f = 15 cm
Image distance = v
The value of v can be obtained using the mirror formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$
$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$
$$= \frac{1}{15} + \frac{1}{12} = \frac{4+5}{60} = \frac{9}{60}$$
$$\therefore v = \frac{60}{9} = 6.7 \text{ cm}$$

Hence, the image of the needle is 6.7 cm away from the mirror. Also, it is on the other side of the mirror.

The image size is given by the magnification formula:

$$m = \frac{h_2}{h_1} = -\frac{v}{u}$$

$$\therefore h_2 = -\frac{v}{u} \times h_1$$

$$= \frac{-6.7}{-12} \times 4.5 = +2.5 \text{ cm}$$

Hence, magnification of the image, $m = \frac{h_2}{h_1} = \frac{2.5}{4.5} = 0.56$

The height of the image is 2.5 cm. The positive sign indicates that the image is erect, virtual, and diminished.
If the needle is moved farther from the mirror, the image will also move away from the mirror, and the size of the image will reduce gradually.

Question 9.3:

A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

Answer 9.3:

Actual depth of the needle in water, h_1 = 12.5 cm and apparent depth of the needle in water, h_2 = 9.4 cm
Refractive index of water = μ
The value of μcan be obtained as follows:

$$\mu = \frac{h_1}{h_2}$$

$$= \frac{12.5}{9.4} \approx 1.33$$

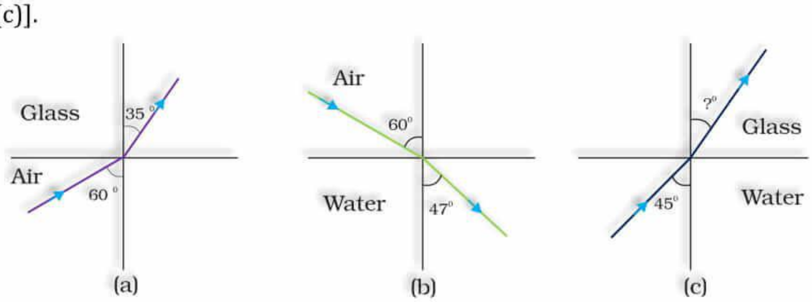
Hence, the refractive index of water is about 1.33.
Water is replaced by a liquid of refractive index, $\mu' = 1.63$
The actual depth of the needle remains the same, but its apparent depth changes.
Let y be the new apparent depth of the needle. Hence, we can write the relation: $\mu' = \frac{h_1}{y}$

$$\Rightarrow y = \frac{h_1}{\mu'} = \frac{12.5}{1.63} = 7.67\ cm$$

Hence, the new apparent depth of the needle is 7.67 cm. It is less than h_2.
Therefore, to focus the needle again, the microscope should be moved up.
∴ Distance by which the microscope should be moved up = 9.4 − 7.67 = 1.73 cm

Question 9.4:

Figures 9.34(a) and (b) show refraction of a ray in air incident at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45º with the normal to a water-glass interface [Fig. 9.34(c)].



Answer 9.4:

As per the given figure, for the glass − air interface:
Angle of incidence, i = 60°
Angle of refraction, r = 35°
The relative refractive index of glass with respect to air is given by Snell's law as:

$$\mu_g^a = \frac{\sin i}{\sin r}$$

$$= \frac{\sin 60°}{\sin 35°} = \frac{0.8660}{0.5736} = 1.51 \qquad \ldots (1)$$

As per the given figure, for the air − water interface:
Angle of incidence, i = 60°
Angle of refraction, r = 47°
The relative refractive index of water with respect to air is given by Snell's law as:

$$\mu_w^a = \frac{\sin i}{\sin r}$$

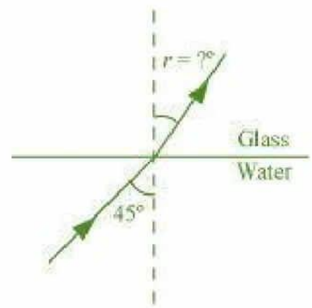
$$= \frac{\sin 60}{\sin 47} = \frac{0.8660}{0.7314} = 1.184 \qquad \ldots (2)$$

Using (1) and (2), the relative refractive index of glass with respect to water can be obtained as:

$$\mu_g^w = \frac{\mu_g^a}{\mu_w^a}$$

$$= \frac{1.51}{1.184} = 1.275$$

The following figure shows the situation involving the glass − water interface.



Angle of incidence, i = 45°
Angle of refraction = r
From Snell's law, r can be calculated as:

$$\frac{\sin i}{\sin r} = \mu_g^w$$

$$\frac{\sin 45°}{\sin r} = 1.275$$

$$\sin r = \frac{\frac{1}{\sqrt{2}}}{1.275} = 0.5546$$

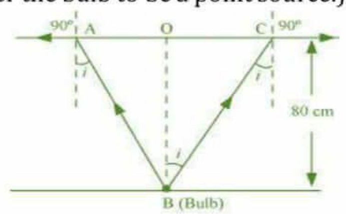
$$\therefore r = \sin^{-1}(0.5546) = 38.68°$$

Hence, the angle of refraction at the water − glass interface is 38.68°.

Question 9.5:

A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)

Answer 9.5:



Actual depth of the bulb in water, d1 = 80 cm = 0.8 m
Refractive index of water, $\mu = 1.33$
The given situation is shown in the following figure:
Where, i = Angle of incidence r = Angle of refraction = 90°
Since the bulb is a point source, the emergent light can be considered as a circle of radius,
$R = \frac{AC}{2} = OA = OB$
Using Snell' law, we can write the relation for the refractive index of water as:

$$\mu = \frac{\sin r}{\sin i}$$

$$1.33 = \frac{\sin 90°}{\sin i}$$

$$\therefore i = \sin^{-1}\left(\frac{1}{1.33}\right) = 48.75°$$

Using the given figure, we have the relation: $\tan i = \frac{OC}{OB} = \frac{R}{d_1}$

∴ R = tan 48.75° × 0.8 = 0.91 m

∴ Area of the surface of water = $\pi R^2 = \pi (0.91)^2 = 2.61\ m^2$

Hence, the area of the surface of water through which the light from the bulb can emerge is approximately 2.61 m^2.

Question 9.6:

A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40°. What is the refractive index of the material of the prism? The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.

Answer 9.6:

Angle of minimum deviation, $\delta_m = 40°$ and angle of the prism, A = 60°

Refractive index of water, μ = 1.33 and refractive index of the material of the prism = μ'

The angle of deviation is related to refractive index (μ') as:

$$\mu' = \frac{\sin\frac{(A+\delta_m)}{2}}{\sin\frac{A}{2}}$$

$$= \frac{\sin\frac{(60° + 40°)}{2}}{\sin\frac{60°}{2}} = \frac{\sin 50°}{\sin 30°} = 1.532$$

Hence, the refractive index of the material of the prism is 1.532.

Since the prism is placed in water, let δ_m' be the new angle of minimum deviation for the same prism.

The refractive index of glass with respect to water is given by the relation:

$$\mu_g^w = \frac{\mu'}{\mu} = \frac{\sin\frac{(A+\delta_m')}{2}}{\sin\frac{A}{2}}$$

$$\sin\frac{(A+\delta_m')}{2} = \frac{\mu'}{\mu}\sin\frac{A}{2}$$

$$\sin\frac{(A+\delta_m')}{2} = \frac{1.532}{1.33} \times \sin\frac{60°}{2} = 0.5759$$

$$\frac{(A+\delta_m')}{2} = \sin^{-1} 0.5759 = 35.16°$$

$$60° + \delta_m' = 70.32°$$

$$\therefore \delta_m' = 70.32° - 60° = 10.32°$$

Hence, the new minimum angle of deviation is 10.32°.

Question 9.7:
Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20 cm?

Answer 9.7:
Refractive index of glass, $\mu = 1.55$
Focal length of the double-convex lens, f = 20 cm
Radius of curvature of one face of the lens = R1
Radius of curvature of the other face of the lens = R2
Radius of curvature of the double-convex lens = R
$\therefore R_1 = R$ and $R_2 = -R$
The value of R can be calculated as:

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{20} = (1.55 - 1)\left[\frac{1}{R} + \frac{1}{R}\right]$$

$$\frac{1}{20} = 0.55 \times \frac{2}{R}$$

$$\therefore R = 0.55 \times 2 \times 20 = 22 \text{ cm}$$

Hence, the radius of curvature of the double-convex lens is 22 cm.

Question 9.8:
A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20 cm, and (b) a concave lens of focal length 16 cm?

Answer 9.8:
In the given situation, the object is virtual and the image formed is real.
Object distance, u = +12 cm
(a) Focal length of the convex lens, f = 20 cm
Image distance = v
According to the lens formula, we have the relation:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{12} = \frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{20} + \frac{1}{12} = \frac{3+5}{60} = \frac{8}{60}$$

$$\therefore v = \frac{60}{8} = 7.5 \text{ cm}$$

Hence, the image is formed 7.5 cm away from the lens, toward its right.

(b) Focal length of the concave lens, f = −16 cm
Image distance = v
According to the lens formula, we have the relation:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = -\frac{1}{16} + \frac{1}{12} = \frac{-3+4}{48} = \frac{1}{48}$$

$$\therefore v = 48 \text{ cm}$$

Hence, the image is formed 48 cm away from the lens, toward its right.

Question 9.9:
An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

Answer 9.9:
Size of the object, h1 = 3 cm
Object distance, u = −14 cm
Focal length of the concave lens, f = −21 cm
Image distance = v
According to the lens formula, we have the relation:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = -\frac{1}{21} - \frac{1}{14} = \frac{-2-3}{42} = \frac{-5}{42}$$

$$\therefore v = -\frac{42}{5} = -8.4 \text{ cm}$$

Hence, the image is formed on the other side of the lens, 8.4 cm away from it. The negative sign shows that the image is erect and virtual.
The magnification of the image is given as:

$$m = \frac{\text{Image height}(h_2)}{\text{Object height}(h_1)} = \frac{v}{u}$$

$$\therefore h_2 = \frac{-8.4}{-14} \times 3 = 0.6 \times 3 = 1.8 \text{ cm}$$

Hence, the height of the image is 1.8 cm.
If the object is moved further away from the lens, then the virtual image will move toward the focus of the lens, but not beyond it. The size of the image will decrease with the increase in the object distance.

Question 9.10:
What is the focal length of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm? Is the system a converging or a diverging lens?
Ignore thickness of the lenses.

Answer 9.10:
Focal length of the convex lens, f_1 = 30 cm and focal length of the concave lens, f_2 = −20 cm
Focal length of the system of lenses = f
The equivalent focal length of a system of two lenses in contact is given as:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60} = -\frac{1}{60}$$

$$\therefore f = -60 \text{ cm}$$

Hence, the focal length of the combination of lenses is 60 cm. The negative sign indicates that the system of lenses acts as a diverging lens.

Question 9.11:
A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision (25 cm), and (b) at infinity? What is the magnifying power of the microscope in each case?

Answer 9.11:
Focal length of the objective lens, f_1 = 2.0 cm and focal length of the eyepiece, f_2 = 6.25 cm
Distance between the objective lens and the eyepiece, d = 15 cm

(a) Least distance of distinct vision, $d' = 25$ cm

∴Image distance for the eyepiece, $v_2 = -25$ cm and let the object distance for the eyepiece = u_2

According to the lens formula, we have the relation:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\frac{1}{u_2} = \frac{1}{v_2} - \frac{1}{f_2}$$

$$= \frac{1}{-25} - \frac{1}{6.25} = \frac{-1-4}{25} = \frac{-5}{25}$$

$$\therefore u_2 = -5 \text{ cm}$$

Image distance for the objective lens, $v_1 = d + u_2 = 15 - 5 = 10$ cm

Object distance for the objective lens = u_1

According to the lens formula, we have the relation:

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\frac{1}{u_1} = \frac{1}{v_1} - \frac{1}{f_1}$$

$$= \frac{1}{10} - \frac{1}{2} = \frac{1-5}{10} = \frac{-4}{10}$$

$$\therefore u_1 = -2.5 \text{ cm}$$

Magnitude of the object distance, $|u_1|$ = 2.5 cm

The magnifying power of a compound microscope is given by the relation:

$$m = \frac{v_1}{|u_1|}\left(1 + \frac{d'}{f_2}\right)$$

$$= \frac{10}{2.5}\left(1 + \frac{25}{6.25}\right) = 4(1+4) = 20$$

Hence, the magnifying power of the microscope is 20.

(b) The final image is formed at infinity.

∴Image distance for the eyepiece, $v_2 = \infty$ and let the object distance for the eyepiece = u_2

According to the lens formula, we have the relation:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\frac{1}{\infty} - \frac{1}{u_2} = \frac{1}{6.25}$$

$$\therefore u_2 = -6.25 \text{ cm}$$

Image distance for the objective lens, $v_1 = d + u_1 = 15 - 6.25 = 8.75$ cm

Object distance for the objective lens = u_1

According to the lens formula, we have the relation:

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\frac{1}{u_1} = \frac{1}{v_1} - \frac{1}{f_1}$$

$$= \frac{1}{8.75} - \frac{1}{2.0} = \frac{2-8.75}{17.5}$$

$$\therefore u_1 = -\frac{17.5}{6.75} = -2.59 \text{ cm}$$

Magnitude of the object distance, $|u_1|$ = 2.59 cm

The magnifying power of a compound microscope is given by the relation:

$$m = \frac{v_1}{|u_1|}\left(\frac{d'}{|u_2|}\right)$$

$$= \frac{8.75}{2.59} \times \frac{25}{6.25} = 13.51$$

Hence, the magnifying power of the microscope is 13.51.

Question 9.12:

A person with a normal near point (25 cm) using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5 cm can bring an object placed at 9.0 mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope,

Answer 9.12:

Focal length of the objective lens, f_o = 8 mm = 0.8 cm

Focal length of the eyepiece, f_e = 2.5 cm

Object distance for the objective lens, u_o = −9.0 mm = −0.9 cm

Least distance of distant vision, d = 25 cm

Image distance for the eyepiece, v_e = −d = −25 cm

Object distance for the eyepiece = u_e

Using the lens formula, we can obtain the value of u_e as:

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$= \frac{1}{-25} - \frac{1}{2.5} = \frac{-1-10}{25} = \frac{-11}{25}$$

$$\therefore u_e = -\frac{25}{11} = -2.27 \text{ cm}$$

We can also obtain the value of the image distance for the objective lens (v_0) using the lens formula.

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$\frac{1}{v_o} = \frac{1}{f_o} + \frac{1}{u_o}$$

$$= \frac{1}{0.8} - \frac{1}{0.9} = \frac{0.9-0.8}{0.72} = \frac{0.1}{0.72}$$

$$\therefore v_o = 7.2 \text{ cm}$$

The distance between the objective lens and the eyepiece $= |u_e| + v_0$

$= 2.27 + 7.2$

$= 9.47$ cm

The magnifying power of the microscope is calculated as:

$$\frac{v_o}{|u_o|}\left(1 + \frac{d}{f_e}\right)$$

$$= \frac{7.2}{0.9}\left(1 + \frac{25}{2.5}\right) = 8(1+10) = 88$$

Hence, the magnifying power of the microscope is 88.

Question 9.13:
A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?

Answer 9.13:
Focal length of the objective lens, f_o = 144 cm
Focal length of the eyepiece, f_e = 6.0 cm
The magnifying power of the telescope is given as:

$$m = \frac{f_o}{f_e}$$

$$= \frac{144}{6} = 24$$

The separation between the objective lens and the eyepiece is calculated as: $f_o + f_e$
= 144 + 6 = 150 cm
Hence, the magnifying power of the telescope is 24 and the separation between the objective lens and the eyepiece is 150 cm.

Question 9.14:
(a) A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is the angular magnification of the telescope?
(b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48 × 106 m, and the radius of lunar orbit is 3.8 × 108 m.

Answer 9.14:
Focal length of the objective lens, fo = 15 m = 15 × 102 cm and focal length of the eyepiece, fe = 1.0 cm
(a) The angular magnification of a telescope is given as:

$$\alpha = \frac{f_o}{f_e}$$

$$= \frac{15 \times 10^2}{1.0} = 1500$$

Hence, the angular magnification of the given refracting telescope is 1500.
(b) Diameter of the moon, d = 3.48 × 106 m and radius of the lunar orbit, r_0 = 3.8 × 108 m
Let d' be the diameter of the image of the moon formed by the objective lens.
The angle subtended by the diameter of the moon is equal to the angle subtended by the image.

$$\frac{d}{r_0} = \frac{d'}{f_o}$$

$$\frac{3.48 \times 10^6}{3.8 \times 10^8} = \frac{d'}{15}$$

$$\therefore d' = \frac{3.48}{3.8} \times 10^{-2} \times 15$$

$$= 13.74 \times 10^{-2} \text{ m} = 13.74 \text{ cm}$$

Hence, the diameter of the moon's image formed by the objective lens is 13.74 cm

Question 9.15:
Use the mirror equation to deduce that:
(a) an object placed between f and 2f of a concave mirror produces a real image beyond 2f.
(b) a convex mirror always produces a virtual image independent of the location of the object.
(c) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.
(d) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.
[Note: This exercise helps you deduce algebraically properties of images that one obtains from explicit ray diagrams.]

Answer 9.15:

(a) For a concave mirror, the focal length (f) is negative. ∴ f < 0
When the object is placed on the left side of the mirror, the object distance (u) is negative. ∴ u < 0
For image distance v, we can write the lens formula as:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u} \qquad \dots(1)$$

The object lies between f and 2f.

$$\therefore 2f<u<f \qquad (\because\ u \text{ and } f \text{ are negative})$$

$$\frac{1}{2f}>\frac{1}{u}>\frac{1}{f}$$

$$-\frac{1}{2f}<-\frac{1}{u}<-\frac{1}{f}$$

$$\frac{1}{f}-\frac{1}{2f}<\frac{1}{f}-\frac{1}{u}<0 \qquad \dots(2)$$

Using equation (1), we get:

$$\frac{1}{2f}<\frac{1}{v}<0$$

Therefore, $\frac{1}{v}$ is negative, i.e., v is negative.

$$\frac{1}{2f}<\frac{1}{v}\Rightarrow 2f>v \quad \Rightarrow -v>-2f$$

Therefore, the image lies beyond 2f.

(b) For a convex mirror, the focal length (f) is positive. ∴ f > 0
When the object is placed on the left side of the mirror, the object distance (u) is negative. ∴ u < 0
For image distance v, we have the mirror formula:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

Using equation (2), we can conclude that:

$$\frac{1}{v}<0$$

$$v>0$$

Thus, the image is formed on the back side of the mirror.
Hence, a convex mirror always produces a virtual image, regardless of the object distance.

(c) For a convex mirror, the focal length (f) is positive. ∴ f > 0
When the object is placed on the left side of the mirror, the object distance (u) is negative, ∴ u < 0
For image distance v, we have the mirror formula:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

But we have $u<0$

$$\therefore \frac{1}{v}>\frac{1}{f}$$

$$v<f$$

Hence, the image formed is diminished and is located between the focus (f) and the pole.

(d) For a concave mirror, the focal length (f) is negative. ∴ f < 0
When the object is placed on the left side of the mirror, the object distance (u) is negative. ∴ u < 0
It is placed between the focus (f) and the pole.

$\therefore f > u > 0$

$\frac{1}{f} < \frac{1}{u} < 0$

$\frac{1}{f} - \frac{1}{u} < 0$

For image distance v, we have the mirror formula:

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$\therefore \frac{1}{v} < 0$

$v > 0$

The image is formed on the right side of the mirror. Hence, it is a virtual image.
For u < 0 and v > 0, we can write: $\frac{1}{u} > \frac{1}{v} \Rightarrow v > u$
Magnification, $m = \frac{v}{u} > 1$
Hence, the formed image is enlarged.

Question 9.16:
A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

Answer 9.16:
Actual depth of the pin, d = 15 cm
Apparent depth of the pin = d'
Refractive index of glass, $\mu = 1.5$
Ratio of actual depth to the apparent depth is equal to the refractive index of glass, i.e.

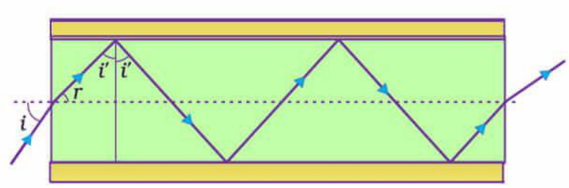
$\mu = \frac{d}{d'}$

$\therefore d' = \frac{d}{\mu}$

$= \frac{15}{1.5} = 10 \text{ cm}$

The distance at which the pin appears to be raised = $d' - d = 15 - 10 = 5$ cm
For a small angle of incidence, this distance does not depend upon the location of the slab.

Question 9.17:
(a) Figure 9.35 shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure.
(b) What is the answer if there is no outer covering of the pipe?



Answer 9.17:

(a) Refractive index of the glass fibre, $\mu_1 = 1.68$
Refractive index of the outer covering of the pipe, $\mu_2 = 1.44$
Angle of incidence = i
Angle of refraction = r
Angle of incidence at the interface = i'
The refractive index (μ) of the inner core – outer core interface is given as:

$$\mu = \frac{\mu_2}{\mu_1} = \frac{1}{\sin i'}$$

$$\sin i' = \frac{\mu_1}{\mu_2}$$

$$= \frac{1.44}{1.68} = 0.8571$$

$$\therefore i' = 59°$$

For the critical angle, total internal reflection (TIR) takes place only when $i > i'$, i.e., $i > 59°$
Maximum angle of reflection,
$r_{max} = 90° - i' = 90° - 59° = 31°$
Let, i_{max} be the maximum angle of incidence.
The refractive index at the air – glass interface, $\mu_1 = 1.68$
We have the relation for the maximum angles of incidence and reflection as:

$$\mu_1 = \frac{\sin i_{max}}{\sin r_{max}}$$

$$\sin i_{max} = \mu_1 \sin r_{max}$$
$$= 1.68 \sin 31°$$
$$= 1.68 \times 0.5150$$
$$= 0.8652$$

$$\therefore i_{max} = \sin^{-1} 0.8652 \approx 60°$$

Thus, all the rays incident at angles lying in the range 0 < i < 60° will suffer total internal reflection.

(b) If the outer covering of the pipe is not present, then:
Refractive index of the outer pipe, μ_1 = Refractive index of air = 1
For the angle of incidence i = 90°,
We can write Snell's law at the air – pipe interface as:

$$\frac{\sin i}{\sin r} = \mu_2 = 1.68$$

$$\sin r = \frac{\sin 90°}{1.68} = \frac{1}{1.68}$$

$$r = \sin^{-1}(0.5952)$$
$$= 36.5°$$
$$\therefore i' = 90° - 36.5° = 53.5°$$

Since $i' > r$, all incident rays will suffer total internal reflection.