# Physics

(www.tiwariacademy.in)

(Chapter 13)(Nuclei)

(Class 12)

## Exercises

**Question 13.1:**

**(a)** Two stable isotopes of lithium ${}^{6}_{3}Li$ and ${}^{7}_{3}Li$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.

**(b)** Boron has two stable isotopes, ${}^{10}_{5}B$ and ${}^{11}_{5}B$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundances of ${}^{10}_{5}B$ and ${}^{11}_{5}B$.

**Answer 13.1:**

**(a)** Mass of ${}^{6}_{3}Li$ lithium isotope, $m_1 = 6.01512$ u

Mass of ${}^{7}_{3}Li$ lithium isotope, $m_2 = 7.01600$ u

Abundance of ${}^{6}_{3}Li$, $\eta_1 = 7.5\%$

Abundance of ${}^{7}_{3}Li$, $\eta_2 = 92.5\%$

The atomic mass of lithium atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2} = \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{7.5 + 92.5} = 6.940934\,u$$

**(b)** Mass of ${}^{10}_{5}B$ boron isotope, $m_1 = 10.01294$ u

Mass of ${}^{11}_{5}B$ boron isotope, $m_2 = 11.00931$ u

Let the abundance of ${}^{10}_{5}B$, $\eta_1 = x\,\%$

Therefore, the abundance of ${}^{11}_{5}B$, $\eta_2 = (100 - x)\,\%$

Atomic mass of boron = 10.811 u. The atomic mass of lithium atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$$

$$\Rightarrow 10.811 = \frac{10.01294 \times x + 11.00931 \times (100 - x)}{x + (100 - x)}$$

$$\Rightarrow 1081.1 = 10.01294x + 1100.931 + 1100.931x$$

$$\Rightarrow x = \frac{19.821}{0.99637} = 19.89\,\%$$

Therefore, $100 - x = 100\,\% - 19.89\,\% = 80.11\,\%$

Hence, the abundances of ${}^{10}_{5}B$ is 19.89 % and ${}^{11}_{5}B$ is 80.11%.

**Question 13.2:**

The three stable isotopes of neon: ${}^{20}_{10}Ne$, ${}^{21}_{10}Ne$ and ${}^{22}_{10}Ne$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

**Answer 13.2:**

Atomic mass of ${}^{20}_{10}Ne$, $m_1 = 19.99$ u

Abundance of ${}^{20}_{10}Ne$, $\eta_1 = 90.51\%$

Atomic mass of ${}^{21}_{10}Ne$, $m_2 = 20.99$ u

Abundance of ${}^{21}_{10}Ne$, $\eta_2 = 0.27\%$

Atomic mass of ${}^{22}_{10}Ne$, $m_3 = 21.99$ u

Abundance of ${}^{22}_{10}Ne$, $\eta_3 = 9.22\%$

The average atomic mass of neon is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3} = \frac{19.99 \times 90.51 + 20.99 \times 0.27 + 21.99 \times 9.22}{90.51 + 0.27 + 9.22} = 20.1771\,u$$

**Question 13.3:**

Obtain the binding energy (in MeV) of a nitrogen nucleus ${}^{14}_{7}N$, given $m({}^{14}_{7}N)$ =14.00307 u.

**Answer 13.3:**

Atomic mass of ${}^{14}_{7}N$ nitrogen, m = 14.00307 u

A nucleus of ${}^{14}_{7}N$ nitrogen contains 7 protons and 7 neutrons.

Hence, the mass defect of this nucleus, $\Delta m = 7m_H + 7m_n - m$

Where,

Mass of a proton, $m_H = 1.007825$ u

Mass of a neutron, $m_n = 1.008665$ u

$\therefore \Delta m = 7 \times 1.007825 + 7 \times 1.008665 - 14.00307 = 7.054775 + 7.06055 - 14.00307 = 0.11236$ u

But 1 u = 931.5 MeV/$c^2$

$\therefore \Delta m = 0.11236 \times 931.5$ MeV/$c^2$

Hence, the binding energy of the nucleus is given as:
$E_b = \Delta mc^2$ Where, c = Speed of light
$\therefore E_b = 0.11236 \times 931.5 \left(\frac{MeV}{c^2}\right) c^2 = 104.66334$ MeV
Hence, the binding energy of a nitrogen nucleus is 104.66334 MeV.

### Question 13.4:

Obtain the binding energy of the nuclei $^{56}_{26}Fe$ and $^{209}_{83}Bi$ in units of MeV from the following data:
$m(^{56}_{26}Fe) = 55.934939\ u,$ $\qquad m(^{209}_{83}Bi) = 208.980388\ u$

### Answer 13.4:

Atomic mass of $^{56}_{26}Fe$, $m_1 = 55.934939$ u
$^{56}_{26}Fe$ nucleus has 26 protons and (56 – 26) = 30 neutrons
Hence, the mass defect of the nucleus, $\Delta m = 26 \times m_H + 30 \times m_n - m_1$
Where,
Mass of a proton, $m_H = 1.007825$ u
Mass of a neutron, $m_n = 1.008665$ u
$\therefore \Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939 = 26.20345 + 30.25995 - 55.934939 = 0.528461$ u
But 1 u = 931.5 MeV/$c^2$
$\therefore \Delta m = 0.528461 \times 931.5$ MeV/$c^2$
The binding energy of this nucleus is given as:
$E_{b1} = \Delta mc^2$ Where, c = Speed of light
$\therefore E_{b1} = 0.528461 \times 931.5 \left(\frac{MeV}{c^2}\right) c^2 = 492.26$ MeV
Average binding energy per nucleon $= \frac{492.26}{56} = 8.79\ MeV$
Atomic mass of $^{209}_{83}Bi$, $m_2 = 208.980388$ u

$^{209}_{83}Bi$ nucleus has 83 protons and (209 – 83) 126 neutrons.
Hence, the mass defect of this nucleus is given as:
$\Delta m' = 83 \times m_H + 126 \times m_n - m_2$
Where,
Mass of a proton, $m_H = 1.007825$ u
Mass of a neutron, $m_n = 1.008665$ u
$\therefore \Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388 = 83.649475 + 127.091790 - 208.980388 = 1.760877$ u
But 1 u = 931.5 MeV/$c^2$
$\therefore \Delta m' = 1.760877 \times 931.5$ MeV/$c^2$
Hence, the binding energy of this nucleus is given as:
$E_{b2} = \Delta m' c^2$
$= 1.760877 \times 931.5 \left(\frac{MeV}{c^2}\right) c^2 = 1640.26$ MeV
Average binding energy per nucleon = $\frac{1640.26}{209} = 7.848\ MeV$

### Question 13.5:

A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}Cu$ atoms (of mass 62.92960 u).

### Answer 13.5:

Mass of a copper coin, m' = 3 g
Atomic mass of $^{63}_{29}Cu$ atom, m = 62.92960 u
The total number of $^{63}_{29}Cu$ atoms in the coin, $N = \frac{N_A \times m'}{Mass\ Number}$
Where,
$N_A$ = Avogadro's number = $6.023 \times 10^{23}$ atoms /g and mass number = 63 g

$$N = \frac{6.022 \times 10^{23} \times 3}{63} = 2.868 \times 10^{22} \text{ atoms}$$

$^{63}_{29}Cu$ nucleus has 29 protons and (63 – 29) 34 neutrons.
$\therefore$ Mass defect of this nucleus, $\Delta m' = 29 \times m_H + 34 \times m_n - m$

Where, Mass of a proton, $m_H = 1.007825$ u
Mass of a neutron, $m_n = 1.008665$ u
$\therefore \Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296 = 0.591935$ u
Mass defect of all the atoms present in the coin, $\Delta m = 0.591935 \times 2.868 \times 10^{22} = 1.69766958 \times 10^{22}$ u
But 1 u = 931.5 MeV/$c^2$
$\therefore \Delta m = 1.69766958 \times 10^{22} \times 931.5$ MeV/$c^2$
Hence, the binding energy of the nuclei of the coin is given as:
$E_b = \Delta mc^2 = 1.69766958 \times 10^{22} \times 931.5 \left(\frac{MeV}{c^2}\right) c^2 = 1.581 \times 10^{25}$ MeV
But 1 MeV = $1.6 \times 10^{-13}$ J
$E_b = 1.581 \times 10^{25} \times 1.6 \times 10^{-13} = 2.5296 \times 10^{12}$ J
This much energy is required to separate all the neutrons and protons from the given coin.

## Question 13.6:

Write nuclear reaction equations for
(i) α-decay of $^{226}_{88}Ra$
(ii) α – decay of $^{242}_{94}Pu$
(iii) β−-decay of $^{32}_{15}P$
(iv) β–decay of $^{210}_{83}Bi$
(v) β+-decay of $^{12}_{6}C$
(vi) β -decay of $^{97}_{43}Tc$
(vii) Electron capture of $^{120}_{54}Xe$

## Answer 13.6:

A is a nucleus of helium $^{4}_{2}He$ and β is an electron ($e^-$ for $\beta^-$ and $e^+$ for $\beta^+$). In every α-decay, there is a loss of 2 protons and 4 neutrons. In every $\beta^+$-decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every $\beta^-$-decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:

(i) $${}_{88}Ra^{226} \longrightarrow {}_{86}Rn^{222} + {}_{2}He^{4}$$

(ii) $$^{242}_{94}Pu \longrightarrow {}^{238}_{92}U + {}^{4}_{2}He$$

(iii) $$^{32}_{15}P \longrightarrow {}^{32}_{16}S + e^- + \overline{\nu}$$

(iv) $$^{210}_{83}B \longrightarrow {}^{210}_{84}PO + e^- + \overline{\nu}$$

(v) $$^{11}_{6}C \longrightarrow {}^{11}_{5}B + e^+ + \nu$$

(vi) $$^{97}_{43}Tc \longrightarrow {}^{97}_{42}MO + e^+ + \nu$$

(vii) $$^{120}_{54}Xe + e^+ \longrightarrow {}^{120}_{53}I + \nu$$

## Question 13.7:

A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

## Answer 13.7:

Half-life of the radioactive isotope = T years and original amount of the radioactive isotope = $N_0$
(a) After decay, the amount of the radioactive isotope = N
It is given that only 3.125% of $N_0$ remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 3.125\% = \frac{3.125}{100} = \frac{1}{32}$$

But $\frac{N}{N_0} = e^{-\lambda t}$

Where, λ = Decay constant and t = Time

$$\therefore -\lambda t = \frac{1}{32}$$

$$-\lambda t = \ln 1 - \ln 32$$
$$-\lambda t = 0 - 3.4657$$

$$t = \frac{3.4657}{\lambda}$$

$$\text{Since } \lambda = \frac{0.693}{T}$$

$$\therefore t = \frac{3.466}{\frac{0.693}{T}} \approx 5T \text{ years}$$

Hence, the isotope will take about 5T years to reduce to 3.125% of its original value.

(b) After decay, the amount of the radioactive isotope = N

It is given that only 1% of $N_0$ remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 1\% = \frac{1}{100}$$

$$\text{But } \frac{N}{N_0} = e^{-\lambda t}$$

$$\therefore e^{-\lambda t} = \frac{1}{100}$$

$$-\lambda t = \ln 1 - \ln 100$$

$$-\lambda t = 0 - 4.6052$$

$$t = \frac{4.6052}{\lambda}$$

Since, $\lambda = 0.693/T$

$$\therefore t = \frac{4.6052}{\frac{0.693}{T}} = 6.645T \text{ years}$$

Hence, the isotope will take about 6.645T years to reduce to 1% of its original value.

## Question 13.8:

The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive ${}^{14}_{6}C$ present with the stable carbon isotope ${}^{12}_{6}C$ . When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of ${}^{14}_{6}C$, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of ${}^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

## Answer 13.8:

Decay rate of living carbon-containing matter,

R = 15 decay/min

Let N be the number of radioactive atoms present in a normal carbon- containing matter.

Half-life of ${}^{14}_{6}C, T_{1/2}$ = 5730 years

The decay rate of the specimen obtained from the Mohenjodaro site:

R' = 9 decays/min

Let N' be the number of radioactive atoms present in the specimen during the Mohenjodaro period.

Therefore, we can relate the decay constant, λ and time, t as:

$$\frac{N}{N'} = \frac{R}{R'} = e^{-\lambda t}$$

$$e^{-\lambda t} = \frac{9}{15} = \frac{3}{5}$$

$$-\lambda t = \log_e \frac{3}{5} = -0.5108$$

$$\therefore t = \frac{0.5108}{\lambda}$$

$$\text{But } \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5730}$$

$$\therefore t = \frac{0.5108}{\frac{0.693}{5730}} = 4223.5 \text{ years}$$

Hence, the approximate age of the Indus-Valley civilisation is 4223.5 years.

**Question 13.9:**

Obtain the amount of $^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}Co$ is 5.3 years.

**Answer 13.9:**

The strength of the radioactive source is given as:

$$\frac{dN}{dt} = 8.0 \text{ mCi}$$

$$= 8 \times 10^{-3} \times 3.7 \times 10^{10}$$

$$= 29.6 \times 10^{7} \text{ decay/s}$$

Where, N = Required number of atoms

Half-life of $^{60}_{27}Co, T_{1/2} =$ 5.3 years = 5.3 × 365 × 24 × 60 × 60 = $1.67 \times 10^8$ s

For decay constant λ, we have the rate of decay as:

$$\frac{dN}{dt} = \lambda N$$

$$\text{Where,} \quad \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.67 \times 10^8} \text{ s}^{-1}$$

$$\therefore N = \frac{1}{\lambda}\frac{dN}{dt}$$

$$= \frac{29.6 \times 10^7}{\frac{0.693}{1.67 \times 10^8}} = 7.133 \times 10^{16} \text{ atoms}$$

For $^{60}_{27}Co$, mass of $6.023 \times 10^{23}$ (Avogadro's number) atoms = 60 g

$\therefore$ Mass of $7.133 \times 10^{16}$ atoms $= \frac{60 \times 7.133 \times 10^{16}}{6.023 \times 10^{23}} = 7.106 \times 10^{-6}$ g

Hence, the amount of $^{60}_{27}Co$ necessary for the purpose is $7.106 \times 10^{-6}$ g.

**Question 13.10:**

The half-life of $^{90}_{38}Sr$ is 28 years. What is the disintegration rate of 15 mg of this isotope?

**Answer 13.10:**

Half-life of $^{90}_{38}Sr, t_{1/2} = =$ 28 years

= 28 × 365 × 24 × 60 × 60

= $8.83 \times 10^8$ s

Mass of the isotope, m = 15 mg
90 g of ${}^{90}_{38}Sr$ atom contains $6.023 \times 10^{23}$ (Avogadro's number) atoms.
Therefore, 15 mg of ${}^{90}_{38}Sr$ contains atoms = $\frac{6.023\times10^{23}\times15\times10^{-3}}{90} = 1.0038 \times 10^{20}$
Rate of disintegration, $\frac{dN}{dt} = \lambda N$
Where, $\lambda =$ decay constant = $\frac{0.693}{8.83\times10^{8}} s^{-1}$

Therefore, $\frac{dN}{dt} = \frac{0.693\times1.0038\times10^{20}}{8.83\times10^{8}} = 7.878 \times 10^{10}$ atoms/s

Hence, the disintegration rate of 15 mg of the given isotope is $7.878 \times 10^{10}$ atoms/s.

## Question 13.11:

Obtain approximately the ratio of the nuclear radii of the gold isotope ${}^{197}_{79}Au$ and the silver isotope ${}^{107}_{47}Ag$.

## Answer 13.11:

Nuclear radius of the gold isotope ${}^{197}_{79}Au$ = $R_{Au}$
Nuclear radius of the silver isotope ${}^{107}_{47}Ag$ = $R_{Ag}$
Mass number of gold, $A_{Au}$ = 197
Mass number of silver, $A_{Ag}$ = 107
The ratio of the radii of the two nuclei is related with their mass numbers as:

$$\frac{R_{Au}}{R_{Ag}} = \left(\frac{R_{Au}}{R_{Ag}}\right)^{\frac{1}{3}}$$

$$= \left(\frac{197}{107}\right)^{\frac{1}{3}} = 1.2256$$

Hence, the ratio of the nuclear radii of the gold and silver isotopes is about 1.23.

## Question 13.12:

Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of (a) ${}^{226}_{88}Ra$ and (b) ${}^{220}_{86}Rn$.
Given: $m({}^{226}_{88}Ra)$ = 226.02540 u, $m({}^{222}_{86}Rn)$ = 222.01750 u,
$m({}^{220}_{86}Rn)$ = 220.01137 u, $m({}^{216}_{84}Po)$ = 216.00189 u.

## Answer 13.12:

(a) Alpha particle decay of ${}^{226}_{88}Ra$ emits a helium nucleus. As a result, its mass number reduces to (226 – 4) 222 and its atomic number reduces to (88 – 2) 86. This is shown in the following nuclear reaction.

$${}^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Ra} + {}^{4}_{2}\text{He}$$

Q-value of emitted α-particle = (Sum of initial mass – Sum of final mass) $c^2$
Where, c = Speed of light It is given that:

$$m\left({}^{226}_{88}\text{Ra}\right) = 226.02540 \text{ u}$$

$$m\left({}^{222}_{86}\text{Rn}\right) = 222.01750 \text{ u}$$

$$m\left({}^{4}_{2}\text{He}\right) = 4.002603 \text{ u}$$

Q-value = [226.02540 – (222.01750 + 4.002603)] u $c^2$ = 0.005297 u $c^2$
But 1 u = 931.5 MeV/$c^2$
∴ Q = 0.005297 × 931.5 ≈ 4.94 MeV
Kinetic energy of the α-particle $= \frac{\text{Mass number after decay}}{\text{Mass number before decay}} \times Q = \frac{222}{226} \times 4.94 = 4.85\ MeV$

(b) Alpha particle decay of ${}^{220}_{86}Ra$
${}^{220}_{86}Ra \rightarrow {}^{216}_{84}Po + {}^{4}_{e}He$
It is given that:

Mass of ${}^{220}_{86}Ra$ = 220.01137 u and mass of ${}^{216}_{84}Po$ = 216.00189 u
∴ Q-value = $[220.01137 - (216.00189 + 4.00260)] \times 931.5 \approx 641$ MeV
Kinetic energy of the α-particle = $\left(\frac{220-4}{220}\right) \times 6.41$ = 6.29 MeV

## Question 13.13:

The radionuclide $^{11}C$ decays according to

$${}^{11}_{6}C \rightarrow {}^{11}_{5}B + e^{+} + \nu: \qquad T_{1/2} = 20.3\ min$$

The maximum energy of the emitted positron is 0.960 MeV.
Given the mass values:
$m({}^{11}_{6}C) = 11.011434$ u and $m({}^{11}_{6}B) = 11.009305$ u,
calculate Q and compare it with the maximum energy of the positron emitted

## Answer 13.13:

The given values are:
$m({}^{11}_{6}C) = 11.011434$ u and $m({}^{11}_{6}B) = 11.009305$ u
The given nuclear reaction is:

$${}^{11}_{6}C \rightarrow {}^{11}_{5}B + e^{+} + \nu$$

Half-life of ${}^{11}_{6}C$ nuclei, $T_{1/2} = 20.3\ min$
Maximum energy possessed by the emitted positron = 0.960 MeV
The change in the Q-value (ΔQ) of the nuclear masses of the ${}^{11}_{6}C$

$$\Delta Q = [m'({}^{11}_{6}C) - \{m'({}^{11}_{5}B) + m_e\}]c^2$$

Where, $m_e$ = Mass of an electron or positron = 0.000548 u
c = Speed of light and m' = Respective nuclear masses.
If atomic masses are used instead of nuclear masses, then we have to add 6 $m_e$ in the case of ${}^{11}_{6}C$ and 5 $m_e$ in the case of ${}^{11}_{5}B$.
Hence, equation (1) reduces to:

$$\Delta Q = [m({}^{11}_{6}C) - m({}^{11}_{5}B) - 2m_e]c^2$$

Here, $m({}^{11}_{6}C)$ and $m({}^{11}_{5}B)$ are the atomic masses.
∴ ΔQ = $[11.011434 - 11.009305 - 2 \times 0.000548]\ c^2 = (0.001033\ c^2)$ u
But 1 u = 931.5 Mev/$c^2$
∴ ΔQ = $0.001033 \times 931.5 \approx 0.962$ MeV
The value of Q is almost comparable to the maximum energy of the emitted positron.

## Question 13.14:

The nucleus ${}^{23}_{10}Ne$ decays by $\beta^-$ emission. Write down the $\beta$-decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:
$m({}^{23}_{10}Ne)$ = 22.994466 u
$m({}^{23}_{11}Na)$ = 22.989770 u.

## Answer 13.14:

In $\beta^-$ emission, the number of protons increases by 1, and one electron and an antineutrino are emitted from the parent nucleus.
$\beta^-$ emission of the nucleus ${}^{23}_{10}Ne$

$${}^{23}_{10}Ne \rightarrow {}^{23}_{11}Na + e^{-} + \bar{\nu} + Q$$

It is given that:
Atomic mass of $m({}^{23}_{10}Ne)$ = 22.994466 u
Atomic mass of $m({}^{23}_{11}Na)$ = 22.989770 u
Mass of an electron, $m_e$ = 0.000548 u
Q-value of the given reaction is given as: $Q = \left[m\left({}^{23}_{10}\mathrm{Ne}\right) - \left[m\left({}^{23}_{11}\mathrm{Na}\right) + m_e\right]\right]c^2$

There are 10 electrons in and 11 electrons in ${}^{23}_{11}Na$. Hence, the mass of the electron is cancelled in the Q-value equation.
Therefore, $Q = [22.994466 - 22.989770]c^2 = (0.004696\ c^2)\ u$
But 1 u = 931.5 Mev/$c^2$
∴ ΔQ = $0.004696 \times 931.5$ = 4.374 MeV
The daughter nucleus is too heavy as compared to $e^-$ and $\bar{\nu}$. Hence, it carries negligible energy. The kinetic energy of the antineutrino is nearly zero. Hence, the maximum kinetic energy of the emitted electrons is almost equal to the Q-value, i.e., 4.374 MeV.

**Question 13.15:**

The Q value of a nuclear reaction A + b → C + d is defined by
$Q = [\, m_A + m_b - m_C - m_d]c^2$, where the masses refer to the respective nuclei.
Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) $^1_1H + ^3_1H \rightarrow ^2_1H + ^2_1H$

(ii) $^{12}_6C + ^{12}_6C \rightarrow ^{20}_{10}Ne + ^4_2He$

Atomic masses are given to be

$m\left(^2_1H\right) = 2.014102 \text{ u}$

$m\left(^3_1H\right) = 3.016049 \text{ u}$

$m\left(^{12}_6C\right) = 12.000000 \text{ u}$

$m\left(^{20}_{10}Ne\right) = 19.992439 \text{ u}$

**Answer 13.15:**

(i) The given nuclear reaction is:

It is given that: ${}_1H^1 + {}^3_1H \rightarrow {}^2_1H + {}^2_1H$

Atomic mass $m\left(^1_1H\right) = 1.007825 \text{ u}$

Atomic mass $m\left(^3_1H\right) = 3.016049 \text{ u}$

Atomic mass $m\left(^2_1H\right) = 2.014102 \text{ u}$

According to the question, the Q-value of the reaction can be written as:

$$Q = \left[m\left(^1_1H\right) + m\left(^3_1H\right) - 2m\left(^2_1H\right)\right]c^2$$

$$= \left[1.007825 + 3.016049 - 2 \times 2.014102\right]c^2$$

$$Q = \left(-0.00433\, c^2\right) \text{ u}$$

But $1 \text{ u} = 931.5 \text{ MeV}/c^2$

$$\therefore Q = -0.00433 \times 931.5 = -4.0334 \text{ MeV}$$

The negative Q-value of the reaction shows that the reaction is endothermic.

(ii) The given nuclear reaction is:

It is given that: $^{12}_6C + ^{12}_6C \rightarrow ^{20}_{10}Ne + ^4_2He$

Atomic mass of $m\left(^{12}_6C\right) = 12.0 \text{ u}$

Atomic mass of $m\left(^{20}_{10}Ne\right) = 19.992439 \text{ u}$

Atomic mass of $m\left(^4_2He\right) = 4.002603 \text{ u}$

The Q-value of this reaction is given as:

$$Q = \left[2m\left({}^{12}_{6}\text{C}\right) - m\left({}^{20}_{10}\text{Ne}\right) - m\left({}^{4}_{2}\text{He}\right)\right]c^2$$

$$= \left[2 \times 12.0 - 19.992439 - 4.002603\right]c^2$$

$$= \left(0.004958\ c^2\right)\ \text{u}$$

$$= 0.004958 \times 931.5 = 4.618377\ \text{MeV}$$

The positive Q-value of the reaction shows that the reaction is exothermic.

**Question 13.16:**

Suppose, we think of fission of a ${}^{56}_{26}Fe$ nucleus into two equal fragments, ${}^{28}_{13}Al$. Is the fission energetically possible? Argue by working out Q of the process. Given $m({}^{56}_{26}Fe) = 55.93494\ u$ and $m({}^{28}_{13}Al) = 27.98191\ u$.

**Answer 13.16:**

The fission of ${}^{56}_{26}Fe$ can be given as:

$${}^{56}_{13}\text{Fe} \longrightarrow 2\,{}^{28}_{13}\text{Al}$$

It is given that:

Atomic mass of $m({}^{56}_{26}Fe) = 55.93494\ u$

Atomic mass of $m({}^{28}_{13}Al) = 27.98191\ u$

The Q-value of this nuclear reaction is given as:

$$Q = \left[m\left({}^{56}_{26}\text{Fe}\right) - 2m\left({}^{28}_{13}\text{Al}\right)\right]c^2$$

$$= \left[55.93494 - 2 \times 27.98191\right]c^2$$

$$= \left(-0.02888\ c^2\right)\ \text{u}$$

But $1\,\text{u} = 931.5\ \text{MeV}/c^2$

$$\therefore Q = -0.02888 \times 931.5 = -26.902\ \text{MeV}$$

The Q-value of the fission is negative. Therefore, the fission is not possible energetically. For an energetically-possible fission reaction, the Q-value must be positive.

**Question 13.17:**

The fission properties of ${}^{239}_{94}Pu$ are very similar to those of ${}^{235}_{92}U$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure ${}^{239}_{94}Pu$ undergo fission?

**Answer 13.17:**

Average energy released per fission of ${}^{239}_{94}Pu$, $E_{av} = 180\ MeV$

Amount of pure ${}^{239}_{94}Pu$, m = 1 kg = 1000 g

$N_A$= Avogadro number = $6.023 \times 10^{23}$

Mass number of ${}^{239}_{94}Pu$ = 239 g

1 mole of ${}^{239}_{94}Pu$ contains $N_A$ atoms.

$\therefore$ m g of ${}^{239}_{94}Pu$ contains $\left(\frac{N_A}{Mass\ number} \times m\right)$ atoms

$\therefore$ 1000 g of ${}^{239}_{94}Pu$ contains $\left(\frac{N_A}{Mass\ number} \times 1000\right)$ atoms $= 2.52 \times 10^{24}$ atoms

$\therefore$ Total energy released during the fission of 1 kg of ${}^{239}_{94}Pu$ is calculated as:

$$E = E_{av} \times 2.52 \times 10^{24}$$

$$= 180 \times 2.52 \times 10^{24} = 4.536 \times 10^{26}\ \text{MeV}$$

Hence, $4.536 \times 10^{26}\ MeV$ is released if all the atoms in 1 kg of pure ${}^{239}_{94}Pu$ undergo fission.

**Question 13.18:**

A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much ${}^{235}_{92}U$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of ${}^{235}_{92}U$ and that this nuclide is consumed only by the fission process.

### Answer 13.18:

Half-life of the fuel of the fission reactor, $t_{1/2} = 5$ years = 5 × 365 × 24 × 60 × 60 s

We know that in the fission of 1 g of $^{235}_{92}U$ nucleus, the energy released is equal to 200 MeV.

1 mole, i.e., 235 g of $^{235}_{92}U$ contains 6.023 × $10^{23}$ atoms.

∴1 g $^{235}_{92}U$ contains $\frac{6.023\times10^{23}}{235}$ atoms.

The total energy generated per gram of $^{235}_{92}U$ is calculated as:

$$E = \frac{6.023\times10^{23}}{235}\times 200 \text{ MeV/g}$$

$$= \frac{200\times6.023\times10^{23}\times1.6\times10^{-19}\times10^{6}}{235} = 8.20\times10^{10}\ \text{J/g}$$

The reactor operates only 80% of the time.

Hence, the amount of $^{235}_{92}U$ consumed in 5 years by the 1000 MW fission reactor is calculated as:

$$= \frac{5\times80\times60\times60\times365\times24\times1000\times10^{6}}{100\times8.20\times10^{10}}\ \text{g}$$

$$\approx 1538 \text{ kg}$$

∴ Initial amount of $^{235}_{92}U$ = 2 × 1538 = 3076 kg

### Question 13.19:

How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as

$$^{2}_{1}\text{H} + ^{2}_{1}\text{H} \longrightarrow ^{3}_{1}\text{He} + \text{n} + 3.27 \text{ MeV}$$

### Answer 13.19:

The given fusion reaction is:

$$^{2}_{1}\text{H} + ^{2}_{1}\text{H} \longrightarrow ^{3}_{2}\text{He} + \text{n} + 3.27 \text{ MeV}$$

Amount of deuterium, m = 2 kg

1 mole, i.e., 2 g of deuterium contains 6.023 × $10^{23}$ atoms.

∴ 2.0 kg of deuterium contains $\frac{6.023\times10^{23}}{2}\times 2000 = 6.023\times10^{26}$ atoms

It can be inferred from the given reaction that when two atoms of deuterium fuse, 3.27 MeV energy is released.

∴Total energy per nucleus released in the fusion reaction:

$$E = \frac{3.27}{2}\times6.023\times10^{26} \text{ MeV}$$

$$= \frac{3.27}{2}\times6.023\times10^{26}\times1.6\times10^{-19}\times10^{6}$$

$$= 1.576\times10^{14} \text{ J}$$

Power of the electric lamp, P = 100 W = 100 J/s

Hence, the energy consumed by the lamp per second = 100 J

The total time for which the electric lamp will glow is calculated as: $\frac{1.576\times10^{14}}{100} s = \frac{1.576\times10^{14}}{100\times60\times60\times24\times365} \approx 4.9\times10^{4}$ years

### Question 13.20:

Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

### Answer 13.20:

When two deuterons collide head-on, the distance between their centres, d is given as:

Radius of 1st deuteron + Radius of 2nd deuteron

Radius of a deuteron nucleus = 2 fm = 2 × $10^{-15}$ m

∴ d = 2 × $10^{-15}$ + 2 × $10^{-15}$ = 4 × $10^{-15}$ m

Charge on a deuteron nucleus = Charge on an electron = e = $1.6 \times 10^{-19}$ C Potential energy of the two-deuteron system:

$$V = \frac{e^2}{4\pi \in_0 d}$$

Where, $\epsilon_0$ = permittivity of free space.

$$\frac{1}{4\pi \in_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\therefore V = \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{4 \times 10^{-15}} \text{ J}$$

$$= \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{4 \times 10^{-15} \times \left(1.6 \times 10^{-19}\right)} \text{ eV}$$

$$= 360 \text{ keV}$$

Hence, the height of the potential barrier of the two-deuteron system is 360 keV.

## Question 13.21:

From the relation $R = R_0A^{1/3}$, where $R_0$ is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

## Answer 13.21:

We have the expression for nuclear radius as:

$R = R_0A^{1/3}$

Where, $R_0$ = Constant

A = Mass number of the nucleus

Nuclear matter density, $\rho = \frac{Mass\ of\ the\ nucleus}{Volume\ of\ the\ nucleus}$

Let m be the average mass of the nucleus. Hence, mass of the nucleus = mA

$$\therefore \rho = \frac{mA}{\frac{4}{3}\pi R^3} = \frac{3mA}{4\pi\left(R_0 A^{\frac{1}{3}}\right)^3} = \frac{3mA}{4\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Hence, the nuclear matter density is independent of A. It is nearly constant.

## Question 13.22:

For the $\beta^+$ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K–shell, is captured by the nucleus and a neutrino is emitted).

$$e^+ + {}^A_ZX \rightarrow {}_{Z-1}^{A}Y + \nu$$

Show that if $\beta^+$ emission is energetically allowed, electron capture is necessarily allowed but not vice–versa.

## Answer 13.22:

Let the amount of energy released during the electron capture process be $Q_1$. The nuclear reaction can be written as:

$$e^+ + {}^A_ZX \rightarrow {}_{Z-1}^{A}Y + \nu + Q_1 \quad \dots (1)$$

Let the amount of energy released during the positron capture process be $Q_2$. The nuclear reaction can be written as:

$${}^A_ZX \rightarrow {}_{Z-1}^{A}Y + e^+ + \nu + Q_2 \quad \dots (2)$$

Where,

$m_N({}^A_ZX)$ = Nuclear mass of ${}^A_ZX$

$m({}^A_ZX)$ = Atomic mass of ${}^A_ZX$.

$m_N({}_{Z-1}^{A}Y)$ = Nuclear mass of ${}_{Z-1}^{A}Y$

$m({}_{Z-1}^{A}Y)$ = Atomic mass of ${}_{Z-1}^{A}Y$

$m_e$ = Mass of an electron

c = Speed of light

Q-value of the electron capture reaction is given as:

$$Q_1 = \left[m_N\left({}^{A}_{Z}X\right) + m_e - m_N\left({}^{A}_{Z-1}Y\right)\right]c^2$$

$$= \left[m\left({}^{A}_{Z}X\right) - Zm_e + m_e - m\left({}^{A}_{Z-1}Y\right) + (Z-1)m_e\right]c^2$$

$$= \left[m\left({}^{A}_{Z}X\right) - m\left({}^{A}_{Z-1}Y\right)\right]c^2 \qquad \dots(3)$$

Q-value of the positron capture reaction is given as:

$$Q_2 = \left[m_N\left({}^{A}_{Z}X\right) - m_N\left({}^{A}_{Z-1}Y\right) - m_e\right]c^2$$

$$= \left[m\left({}^{A}_{Z}X\right) - Zm_e - m\left({}^{A}_{Z-1}Y\right) + (Z-1)m_e - m_e\right]c^2$$

$$= \left[m\left({}^{A}_{Z}X\right) - m\left({}^{A}_{Z-1}Y\right) - 2m_e\right]c^2 \qquad \dots(4)$$

It can be inferred that if $Q_2 > 0$, then $Q_1 > 0$; Also, if $Q_1 > 0$, it does not necessarily mean that $Q_2 > 0$.

In other words, this means that if $\beta^+$ emission is energetically allowed, then the electron capture process is necessarily allowed, but not vice-versa. This is because the Q-value must be positive for an energetically-allowed nuclear reaction.