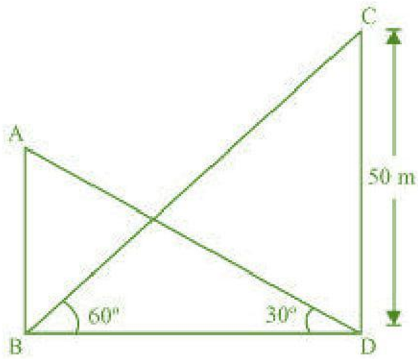
Question 9:
The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Answer 9:


Let CD is 50 m high tower and AB is building.

In ΔCDB,

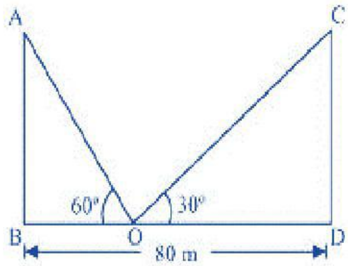
$\frac{CD}{BD} = \tan 60° \quad \Rightarrow \frac{50}{BD} = \sqrt{3} \quad \Rightarrow BD = \frac{50}{\sqrt{3}}$

In ΔABD,

$\frac{AB}{BD} = \tan 30° \quad \Rightarrow \frac{AB}{50/\sqrt{3}} = \frac{1}{\sqrt{3}} \quad \Rightarrow AB = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{50}{3} = 16\frac{2}{3}$

Hence, the height of the building is $16\frac{2}{3}$ m.

Question 10:
Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

Answer 10:


Let BD is 80 m wide road and AB & CD are the two equal poles.

Let $AB = CD = x$

In ΔABO,

$\frac{AB}{BO} = \tan 60° \quad \Rightarrow \frac{x}{BO} = \sqrt{3} \quad \Rightarrow BO = \frac{x}{\sqrt{3}}$

In ΔCDO,

$\frac{CD}{DO} = \tan 30° \quad \Rightarrow \frac{x}{80 - BO} = \frac{1}{\sqrt{3}} \quad \Rightarrow x\sqrt{3} = 80 - BO$

$\Rightarrow x\sqrt{3} = 80 - \frac{x}{\sqrt{3}}$ [Putting the value of BO]

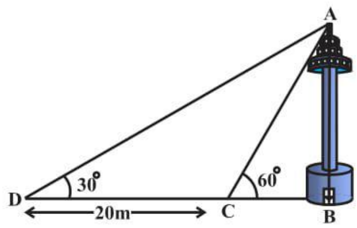
$\Rightarrow x\sqrt{3} + \frac{x}{\sqrt{3}} = 80 \quad \Rightarrow 3x + x = 80\sqrt{3} \quad \Rightarrow 4x = 80\sqrt{3} \quad \Rightarrow x = 20\sqrt{3}$

Therefore, $BO = \frac{x}{\sqrt{3}} = \frac{20\sqrt{3}}{\sqrt{3}} = 20$

DO = BD – BO = (80 – 20) m = 60 m

Hence, the height of the pole is $20\sqrt{3}$ m and distance of poles from the point is 20 m and 60 m.

Question 11:
A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joing this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (see Figure). Find the height of the tower and the width of the canal.



Answer 11:
Let AB is TV tower and BC is width of the canal. In ΔABC,

$\frac{AB}{BC} = \tan 60° \quad \Rightarrow \frac{AB}{BC} = \sqrt{3}$

$\Rightarrow BC = \dfrac{AB}{\sqrt{3}}$

In ΔABD,

$\dfrac{AB}{BD} = \tan 30° \quad \Rightarrow \dfrac{AB}{BC + CD} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \dfrac{AB}{AB/\sqrt{3} + CD} = \dfrac{1}{\sqrt{3}}$ [Putting the value of BC]

$\Rightarrow \dfrac{AB\sqrt{3}}{AB + 20\sqrt{3}} = \dfrac{1}{\sqrt{3}} \quad \Rightarrow 3AB = AB + 20\sqrt{3} \quad \Rightarrow 2AB = 20\sqrt{3} \quad \Rightarrow AB = 10\sqrt{3}$

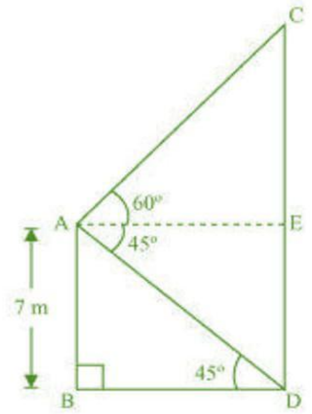
Therefore, $BC = \dfrac{AB}{\sqrt{3}} = \dfrac{10\sqrt{3}}{\sqrt{3}} = 10$

Hence, the height of the tower is $10\sqrt{3}$ m and the width of the tower is 10 m.

Question 12:

From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Answer 12:



Let AB is building and CD is cable tower.

In ΔABD,

$\dfrac{AB}{BD} = \tan 45° \quad \Rightarrow \dfrac{7}{BD} = 1 \quad \Rightarrow BD = 7$

In ΔACE, AC = BD = 7 m

$\dfrac{CE}{AE} = \tan 60° \quad \Rightarrow \dfrac{CE}{7} = \sqrt{3} \quad \Rightarrow CE = 7\sqrt{3}$

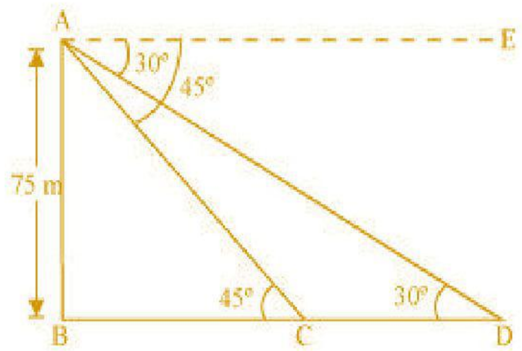
Therefore, $CD = CE + ED = 7\sqrt{3} + 7 = 7(\sqrt{3} + 1)$

Hence, the height of the cable tower is $7(\sqrt{3} + 1)$ m.

Question 13:

As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Answer 13:



Let AB is 75 m high lighthouse and C & D are the two ships.

In ΔABC,

$\dfrac{AB}{BC} = \tan 45° \quad \Rightarrow \dfrac{75}{BC} = 1$

$\Rightarrow BC = 75$

In ΔABD,

$\dfrac{AB}{BD} = \tan 30°$

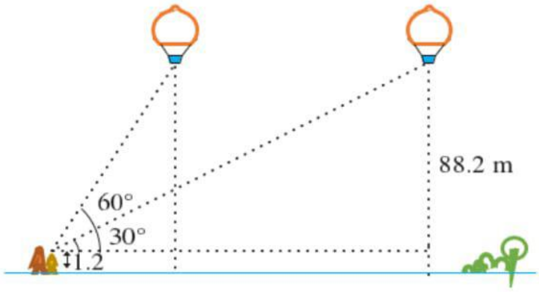
$\Rightarrow \dfrac{75}{BC + CD} = \dfrac{1}{\sqrt{3}} \quad \Rightarrow \dfrac{75}{75 + CD} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow 75 + CD = 75\sqrt{3}$

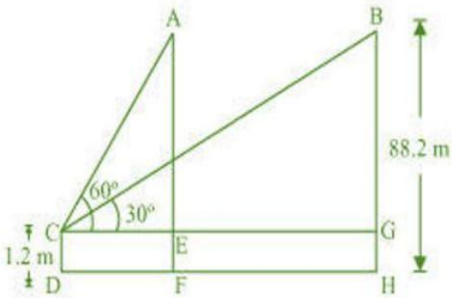
$\Rightarrow CD = 75(\sqrt{3} - 1)$

Hence, the distance between the two ships is $75(\sqrt{3} - 1)$.

Question 14:
A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of
elevation reduces to 30° (see Figure). Find the distance travelled by the balloon during the interval.



Answer 14:
Let CD is 1.2 m high girl and FG is the distance travelled by balloon. In ΔACE,



$\frac{AE}{CE} = \tan 60°$

$\Rightarrow \frac{AF - EF}{CE} = \sqrt{3}$

$\Rightarrow \frac{88.2 - 1.2}{CE} = \sqrt{3}$

$\Rightarrow \frac{87}{CE} = \sqrt{3} \quad \Rightarrow CE = \frac{87}{\sqrt{3}} = \frac{87\sqrt{3}}{3} = 29\sqrt{3}$

In ΔBCG,

$\frac{BG}{GC} = \tan 30° \;\; \Rightarrow \frac{88.2 - 1.2}{CG} = \frac{1}{\sqrt{3}}$

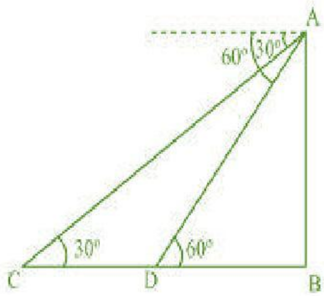
$\Rightarrow \frac{87}{CG} = \frac{1}{\sqrt{3}} \quad \Rightarrow CG = 87\sqrt{3}$

Distance travelled by balloon $= EG = CG - CE = 87\sqrt{3} - 29\sqrt{3} = 58\sqrt{3}$

Hence, the distance travelled by balloon is $58\sqrt{3}$ m.

Question 15:
A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Answer 15:
Let CD is highway and AB is tower. C is the initial position of the car and D is the position after 6 seconds.



In ΔADB,

$\frac{AB}{DB} = \tan 60° \;\; \Rightarrow \frac{AB}{DB} = \sqrt{3} \quad \Rightarrow DB = \frac{AB}{\sqrt{3}}$

In ΔABC,

$\frac{AB}{BC} = \tan 30°$

$\Rightarrow \frac{AB}{BD + DC} = \frac{1}{\sqrt{3}}$

$\Rightarrow AB\sqrt{3} = BD + DC$

$\Rightarrow AB = \dfrac{AB}{\sqrt{3}} + DC$ [Putting the value of BD]

$\Rightarrow AB\sqrt{3} - \dfrac{AB}{\sqrt{3}} = DC$

$\Rightarrow DC = \dfrac{3AB - AB}{\sqrt{3}} = \dfrac{2AB}{\sqrt{3}}$

The time taken to travelled CD $\left(= \frac{2AB}{\sqrt{3}}\right)$ distance = 6 seconds

Therefore, the speed of the car $= \frac{Distance}{Time} = \frac{\left(\frac{2AB}{\sqrt{3}}\right)}{6} = \frac{AB}{3\sqrt{3}}\ m/s$

The time taken to travelled BD $\left(= \frac{AB}{\sqrt{3}}\right)$ distance $= \dfrac{Distance}{speed} = \dfrac{\left(\frac{AB}{\sqrt{3}}\right)}{\frac{AB}{3\sqrt{3}}} = 3$ seconds

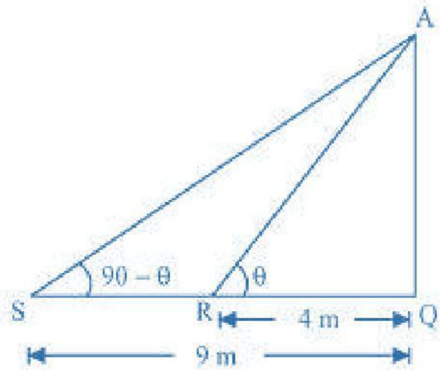
Hence, car will take 3 seconds to reach the foot of the car.

Question 16:

The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Answer 16:

Let AQ is tower and C & D are the two point 4 m & 9 m away from the foot of tower.
The angle of elevations are complementary. So, if one angle is θ, then the other will be 90 – θ.



In ΔAQR,

$\dfrac{AQ}{QR} = \tan\theta$

$\Rightarrow \dfrac{AQ}{4} = \tan\theta$ ... (1)

In ΔAQS,

$\dfrac{AQ}{QS} = \tan(90 - \theta)$

$\Rightarrow \dfrac{AQ}{9} = \cot\theta$ ... (2)

Multiplying the equations (1) and (2), we have

$\left(\dfrac{AQ}{4}\right)\left(\dfrac{AQ}{9}\right) = \tan\theta . \cot\theta$

$\Rightarrow \dfrac{AQ^2}{36} = 1$

$\Rightarrow AQ^2 = 36$

$\Rightarrow AQ = \pm 6$

Height can't be negative. Hence, the height of the tower is 6 m.