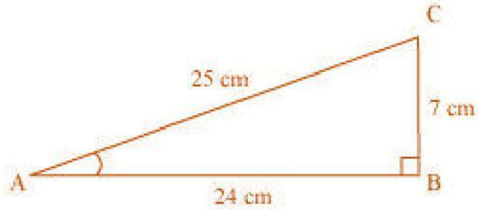
Exercise 8.1

Question 1:

In ΔABC, right angled at B, AB = 24cm, BC = 7cm. Determine:

(i) $\sin A$, $\cos A$ **(ii)** $\sin C$, $\cos C$

Answer 1:



In ΔABC, by Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2 = (24\ cm)^2 + (7\ cm)^2$

$= (576 + 49)\ cm^2 = 625\ cm^2$

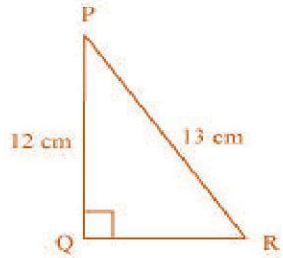
$\Rightarrow AC = \sqrt{625} = 25\ cm$

(i) $\sin A = \frac{BC}{AC} = \frac{7}{25}$ and $\cos A = \frac{AB}{AC} = \frac{24}{25}$

(ii) $\sin C = \frac{AB}{AC} = \frac{24}{25}$ and $\cos C = \frac{BC}{AC} = \frac{7}{25}$

Question 2:

In figure, find $\tan P - \cot R$.



Answer 2:

In ΔPQR, by Pythagoras theorem, we have

$QR^2 = PR^2 - PQ^2 = (13)^2 - (12)^2 = 169 - 144 = 25$

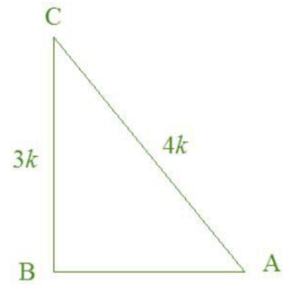
$\Rightarrow QR = \sqrt{25} = 5$

Hence, $\tan P - \cot R = \frac{QR}{PQ} - \frac{QR}{PQ} = \frac{5}{12} - \frac{5}{12} = 0$

Question 3:

If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Answer 3:



Given that: $\sin A = \frac{3}{4}$

Let $\sin A = \frac{3k}{4k}$, where k is a real number.

In ΔABC, by Pythagoras theorem, we have

$AB^2 = AC^2 - BC^2 = (4k)^2 - (3k)^2 = 16k^2 - 9k^2 = 7k^2$

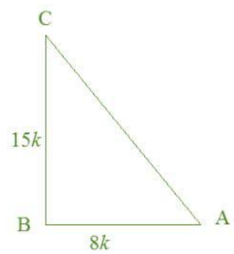
$\Rightarrow AB = \sqrt{7k^2} = \sqrt{7}k$

Hence, $\cos A = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$ and $\tan A = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$

Question 4:

Given $15 \cot A = 8$, find $\sin A$ and $\sec A$.

Answer 4:



Given that: $15 \cot A = 8 \Rightarrow \cot A = \frac{8}{15}$

Let $\cot A = \frac{8k}{15k}$, where k is a real number.

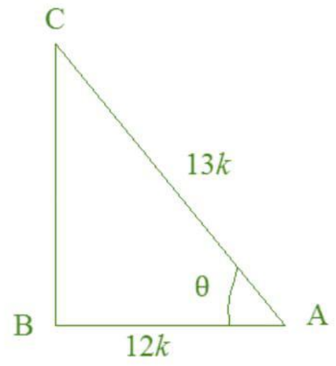
In ΔABC, by Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2 = (8k)^2 + (15k)^2 = 64k^2 + 225k^2 = 289k^2$

$\Rightarrow AC = \sqrt{289k^2} = 17k$

Hence, $\sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$ and $\sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$

Question 5:
Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Answer 5:


Given that: $\sec\theta = \frac{13}{12}$

Let $\sec\theta = \frac{13k}{12k}$, where k is a real number.

In ΔABC, by Pythagoras theorem, we have

$BC^2 = AC^2 - AB^2 = (13k)^2 - (12k)^2 = 169k^2 - 144k^2 = 25k^2$

$\Rightarrow BC = \sqrt{25k^2} = 5k$

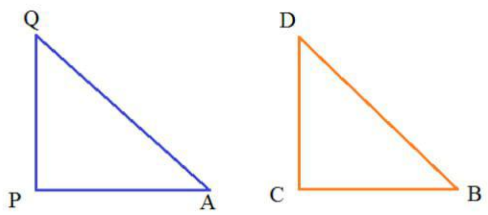
Hence, $\sin\theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$\cos\theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$

$\tan\theta = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$

$\operatorname{cosec}\theta = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$ and $\cot\theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$

Question 6:
If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Answer 6:


Given that: $\cos A = \cos B$

$\cos A = \cos B$

$\Rightarrow \frac{AP}{AQ} = \frac{BC}{BD} \Rightarrow \frac{AP}{BC} = \frac{AQ}{BD}$

Let $\frac{AP}{BC} = \frac{AQ}{BD} = k$ ... (i)

Therefore, $AP = k(BC)$ and $AQ = k(BD)$

Now, in ΔAPQ and ΔBCD

$\frac{PQ}{CD} = \frac{\sqrt{AQ^2 - AP^2}}{\sqrt{BD^2 - BC^2}} = \frac{\sqrt{(k.BD)^2 - (k.BC)^2}}{\sqrt{BD^2 - BC^2}} = \frac{k\sqrt{BD^2 - BC^2}}{\sqrt{BD^2 - BC^2}} = k$ ... (ii)

From the equation (i) and (ii), we get

$\frac{AP}{BC} = \frac{AQ}{BD} = \frac{PQ}{CD}$

So, $\Delta APQ \sim \Delta BCD$ [SSS similarity criteria]

Hence, $\angle A = \angle B$

Question 7:
If $\cot\theta = \frac{7}{8}$, evaluate: **(i)** $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ **(ii)** $\cot^2\theta$

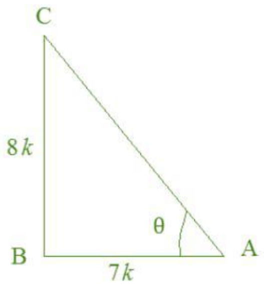
Answer 7:
Given that: $\cot\theta = \frac{7}{8}$

Let $\cot\theta = \frac{7k}{8k}$, where k is a real number.

In ΔABC, by Pythagoras theorem, we have

$AC^2 = BC^2 + AB^2 = (8k)^2 + (7k)^2 = 64k^2 + 49k^2 = 113k^2$

$\Rightarrow AC = \sqrt{113k^2} = \sqrt{113}k$



(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$$= \frac{\left(1+\frac{7}{\sqrt{113}}\right)\left(1-\frac{7}{\sqrt{113}}\right)}{\left(1+\frac{8}{\sqrt{113}}\right)\left(1-\frac{8}{\sqrt{113}}\right)} = \frac{1-\left(\frac{7}{\sqrt{113}}\right)^2}{1-\left(\frac{8}{\sqrt{113}}\right)^2} = \frac{1-\frac{49}{113}}{1-\frac{64}{113}} = \frac{\frac{113-49}{113}}{\frac{113-64}{113}} = \frac{64}{49}$$

(ii) $\cot^2\theta$

$$= (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

Question 8:

If $3\cot A = 4$, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Answer 8:

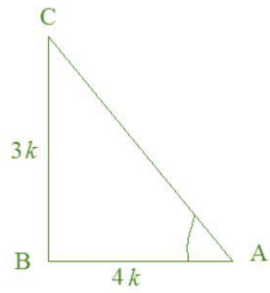
Given that: $3\cot A = 4 \Rightarrow \cot A = \frac{4}{3}$

Let $\cot A = \frac{4k}{3k}$, where k is a real number.

In ΔABC, by Pythagoras theorem, we have

$AC^2 = BC^2 + AB^2 = (3k)^2 + (4k)^2 = 9k^2 + 16k^2 = 25k^2$

$\Rightarrow AC = \sqrt{25k^2} = 5k$



Therefore,

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{7}{25}$$

and

$$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{16-9}{25} = \frac{7}{25}$$

Hence, $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

Question 9:

In triangle ABC, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of:

(i) sin A cos C + cos A sin C **(ii)** cos A cos C – sin A sin C

Answer 9:

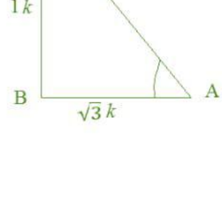
Given that: $\tan A = \frac{1}{\sqrt{3}}$

Let $\tan A = \frac{1k}{\sqrt{3}k}$, where k is a real number.

In ΔABC, by Pythagoras theorem, we have

$AC^2 = BC^2 + AB^2 = (1k)^2 + (\sqrt{3}k)^2 = k^2 + 3k^2 = 4k^2$

$\Rightarrow AC = \sqrt{4k^2} = 2k$



(i) sin A cos C + cos A sin C

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$$

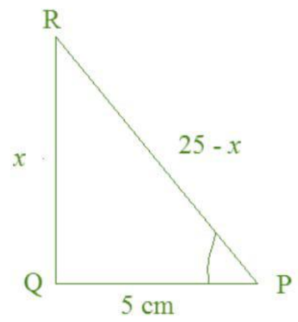
(ii) cos A cos C – sin A sin C

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Question 10:

In ΔPQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of $\sin P, \cos P$ and $\tan P$.

Answer 10:



Given that: in ΔPQR, angle Q is right angled.
Let QR = x, therefore, PR = $25 - x$
In ΔPQR, by Pythagoras theorem, we have
$PR^2 = PQ^2 + OQ^2 \Rightarrow (25 - x)^2 = (5)^2 + (x)^2$
$\Rightarrow 625 + x^2 - 50x = 25 + x^2 \Rightarrow 625 - 50x = 25$
$\Rightarrow 50x = 600 \Rightarrow x = 12 \Rightarrow$ QR = 12
Therefore, PR = 25 − 12 = 13
Now, $\sin P = \frac{QR}{PR} = \frac{12}{13}$, $\cos P = \frac{PQ}{PR} = \frac{5}{13}$ and $\tan P = \frac{QR}{PQ} = \frac{12}{5}$

Question 11:

State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) $\cos A$ is abbreviation used for the cosecant of angle A.

(iv) $\cot A$ is the product of cot and A.

(v) $\sin \theta = \frac{4}{3}$ for some angle θ.

Answer 11:

(i) False,

Because, $\tan A = \frac{Opposite\ side\ of\ angle\ A}{Adjecent\ side\ of\ angle\ A}$, if opposite side > adjacent side, then the value of $\tan A$ is greater than 1.

(ii) True,

Because, $\sec A = \frac{Hypotenuse}{Adjecent\ side\ of\ angle\ A}$ and we know that hypotenuse is always greater than adjacent side.

(iii) False,

Because, $\cos A$ is used for cosine of angle A.

(iv) False,

Because, $\cot A$ is used for cotangent of angle A.

(v) False,

Because, $\sin \theta = \frac{Opposite\ side\ of\ angle\ A}{Hypotenuse}$, we know that hypotenuse is always greater than opposite side.