Exercise 6.4

Question 1:
Let ΔABC ~ ΔDEF and their areas be, respectively, 64 cm² and 121 cm². If EF = 15.4 cm, find BC.

Answer 1:
Given that, ΔABC ~ ΔDEF, therefore

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

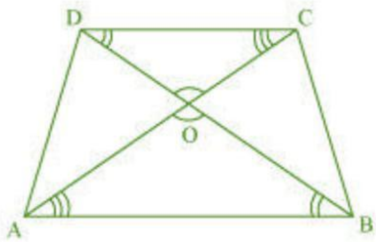
Given: EF = 15.4 cm, ar(ΔABC) = 64 cm² and ar(ΔDEF) = 121 cm², therefore

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$\Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2} \quad \Rightarrow \frac{8}{11} = \frac{BC}{15.4}$

$\Rightarrow \text{BC} = \frac{8 \times 15.4}{11} = 11.2 \text{ cm}$

Question 2:
Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.

Answer 2:


Given: AB || CD,

∴ ∠OAB = ∠OCD and ∠OBA = ∠ODC [Alternate angles]

In ΔAOB and ΔCOD,

∠AOB = ∠COD [Vertically Opposite Angles]

∠OAB = ∠OCD [Alternate angles]

∠OBA = ∠ODC [Alternate angles]

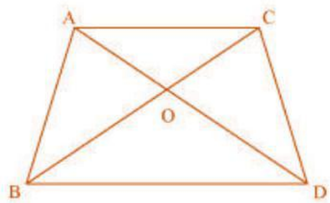
∴ ΔAOB ~ ΔCOD [AAA similarity]

Therefore,

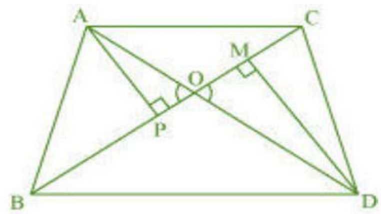
$\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{CD^2}$

$\Rightarrow \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{(2CD)^2}{CD^2}$ [Because AB = 2CD]

$\Rightarrow \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{4CD^2}{CD^2} \qquad \Rightarrow \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{4}{1} = 4:1$

Question 3:


In Figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(\text{ABC})}{ar(\text{DBC})} = \frac{AO}{DO}$.

Answer 3:


Let AP DM are the perpendiculars on BC.

We know that, the area of triangle = $\frac{1}{2}$ × Base × Perpendicular

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AP}{\frac{1}{2} \times BC \times DM} \qquad \dots (1)$$

In ΔAPO and ΔDMO,

∠APO = ∠DMO [Each 90°]

∠AOP = ∠DOM [Vertically Opposite Angles]
∴ ΔAPO ~ ΔDMO [AA similarity]

$$\frac{AP}{DM} = \frac{AO}{DO}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$ [From the equation (1)]

Question 4:

If the areas of two similar triangles are equal, prove that they are congruent.

Answer 4:

Let, ΔABC ~ ΔDEF, therefore

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} \quad \dots (1)$$

Given that, $ar(\Delta ABC) = ar(\Delta DEF)$

Therefore, $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = 1$

From the equation (1), we have, $\frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = 1$

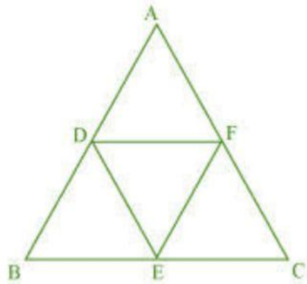
⇒ AB = DE, BC = EF and AC = DF

∴ ΔABC ≅ ΔDEF [SSS congruency theorem]

Question 5:

D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.

Answer 5:



D and E are the mid-points of sides AB and AC respectively.

Therefore, DE || AC and DE = ½ AC

In ΔBED and ΔBCA,

∠BED = ∠BCA [Corresponding Angles]

∠BDE = ∠BAC [Corresponding Angles]

∴ ΔBED ~ ΔBCA [AA similarity]

$$\frac{ar(\Delta BED)}{ar(\Delta BCA)} = \frac{DE^2}{AC^2} \quad \Rightarrow \frac{ar(\Delta BED)}{ar(\Delta BCA)} = \frac{\left(\frac{1}{2}AC\right)^2}{AC^2} \quad \Rightarrow \frac{ar(\Delta BED)}{ar(\Delta BCA)} = \frac{1}{4}$$

$$\Rightarrow ar(\Delta BED) = \frac{1}{4} \times ar(\Delta BCA)$$

Let $ar(\Delta ABC) = x$

Therefore, $ar(\Delta BED) = \frac{1}{4}x$

Similarly,

$ar(\Delta CEF) = \frac{1}{4}x$ and $ar(\Delta ADF) = \frac{1}{4}x$

Now, $ar(\Delta DEF) = ar(\Delta ABC) - ar(\Delta BED) - ar(\Delta CEF) - ar(\Delta ADF)$

$$\Rightarrow ar(\Delta DEF) = x - \frac{1}{4}x - \frac{1}{4}x - \frac{1}{4}x = x - \frac{3}{4}x = \frac{1}{4}x$$

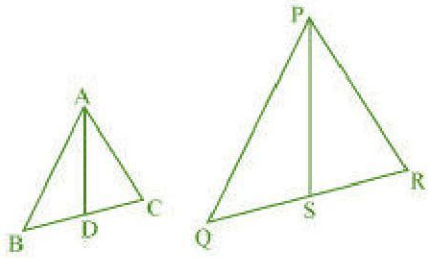
$\Rightarrow ar(\Delta DEF) = \frac{1}{4} \times (\Delta ABC)$ [Because $ar(\Delta ABC) = x$]

$$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

Question 6:

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer 6:



Let ΔABC ~ ΔPQR. AD and PS are the medians of triangle.
Given that, ΔABC ~ ΔPQR

Therefore, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ... (1)

and

∠A = ∠P, ∠B = ∠Q, ∠C = ∠R ... (2)

AD and PS are medians of the triangle. Therefore
∴ BD = DC = ½ BC and QS = SR = ½ QR
From the equation (1), we have

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR}$ ... (3)

In ΔABD and ΔPQS,
∠B = ∠Q [From the equation (2)]
and, $\frac{AB}{PQ} = \frac{BD}{QS}$ [From the equation (3)]
∴ ΔABD ~ ΔPQS [SAS similarity]
Therefore,

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS}$ ... (4)

and

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

Therefore, from the equation (1) and (4), we have

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

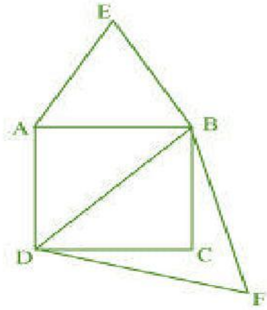
Hence,

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PS^2}$$

Question 7:

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Answer 7:



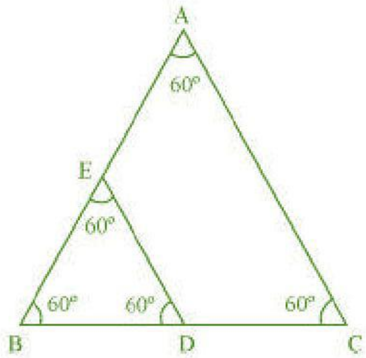
Let ABCD is a square of side a units. Therefore
Diagonal = $\sqrt{2}$a units
The triangles form on side and diagonal are ΔABE and ΔDBF respectively.
The length of each side of triangle ABE = a units and
The length of each side of triangle DBF = $\sqrt{2}$a units
Both the triangles are equilateral and each angle of both the triangles are 60°.
Therefore, by AAA similarity, ΔABE ~ ΔDBF.
Now, using the area theorem, we have

$$\frac{ar(\Delta ABC)}{ar(\Delta DBF)} = \left(\frac{a}{\sqrt{2}a}\right)^2 = \frac{1}{2}$$

Question 8:
ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is

(A) 2:1 (B) 1:2 (C) 4:1 (D) 1:4

Answer 8:
Both the triangles are equilateral and each angle of both the triangles are 60°.



Therefore, by AAA similarity, ΔBCA ~ ΔBDE.

Let, the side of ΔABC = x

Therefore, the side of ΔBDE = x/2

$$\frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \left(\frac{x}{\frac{x}{2}}\right)^2 = \frac{4}{1}$$

Hence, the option (C) is correct.

Question 9:
Sides of two similar triangles are in the ratio 4:9. Areas of these triangles are in the ratio

(A) 2:3 (B) 4:9 (C) 81:16 (D) 16:81

Answer 9:
We know that the ratio of area of similar triangles is equal to the ratio of square of their corresponding sides.

Therefore, the ratio of areas of two triangles = $\left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Hence, the option (D) is correct.