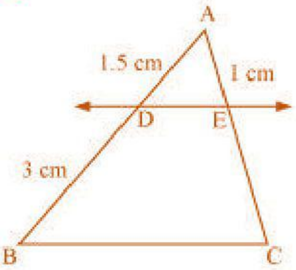
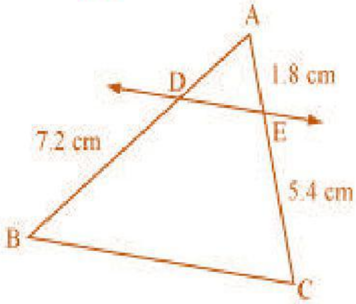
Exercise 6.2

Question 1:
In Figure, (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).

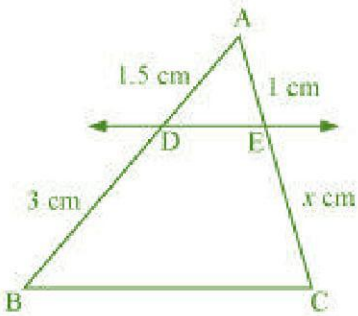
(i)



(ii)



Answer 1:
(i)

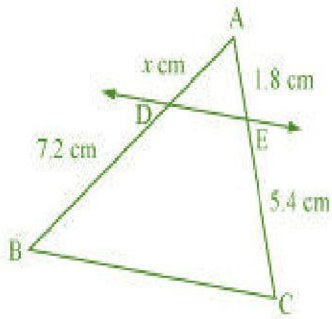


Let EC = x cm
Given that, DE || BC, therefore
Using Thales theorem, we have

$$\frac{AD}{DB} = \frac{AC}{EC}$$
$$\Rightarrow \frac{1.5}{3} = \frac{1}{x}$$
$$\Rightarrow x = \frac{3 \times 1}{1.5} = 2$$

Hence, $EC = 2$.

(ii)



Let AD = x cm
Given that, DE || BC, therefore
Using Thales theorem, we have

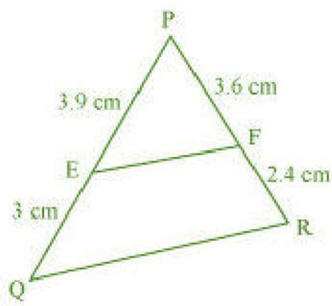
$$\frac{AD}{DB} = \frac{AC}{EC}$$
$$\Rightarrow \frac{x}{7.2} = \frac{1.8}{5.4}$$
$$\Rightarrow x = \frac{1.8 \times 7.2}{5.4} = 2.4$$

Hence, $AD = 2.4$.

Question 2:
E and F are points on the sides PQ and PR respectively of a Δ PQR. For each of the following cases, state whether EF||QR:

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm
(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm
(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.63 cm

Answer 2:
(i)

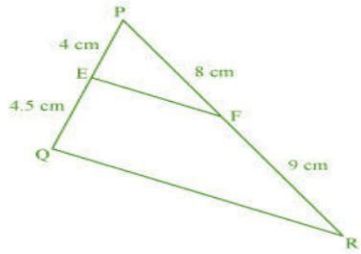


Given that, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm, therefore

$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$ and $\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$

Since, $\frac{PE}{EQ} \neq \frac{PF}{FR}$, Hence, EF is not parallel to QR.

(ii)



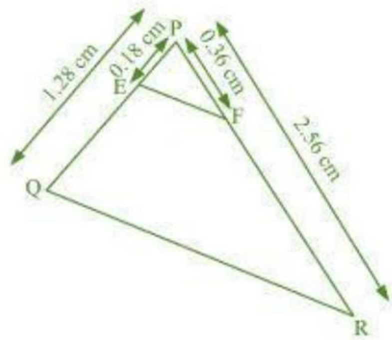
Given that, PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm, therefore

$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$ and $\frac{PF}{FR} = \frac{8}{9}$

Here, $\frac{PE}{EQ} = \frac{PF}{FR}$,

Hence, according to converse of Thales theorem, EF || QR.

(iii)



Given that, PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm, so

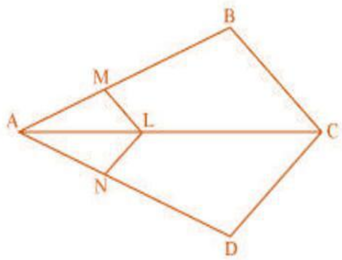
$\frac{PE}{EQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$ and $\frac{PF}{FR} = \frac{0.36}{2.56} = \frac{9}{64}$

Here, $\frac{PE}{EQ} = \frac{PF}{FR}$,

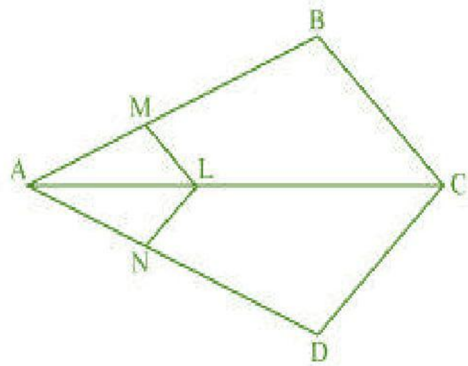
Hence, according to converse of Thales theorem, EF || QR.

Question 3:

In Figure, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



Answer 3:



Given that, in triangle ABC, LM || CB, therefore

According to Thales theorem, we have

$\frac{AM}{AB} = \frac{AL}{AC}$ ... (1)

Similarly,

Given that, in triangle ADC, LN || CD, therefore

According to Thales theorem, we have

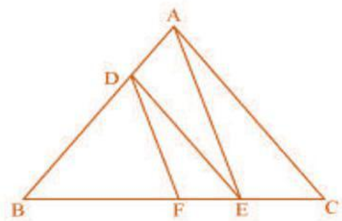
$\frac{AN}{AD} = \frac{AL}{AC}$ ... (2)

From the equation (1) and (2), we have

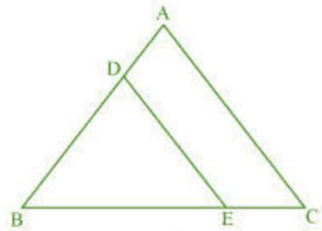
$$\frac{AM}{AB} = \frac{AN}{AD}$$

Question 4:

In Figure, DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.



Answer 4:



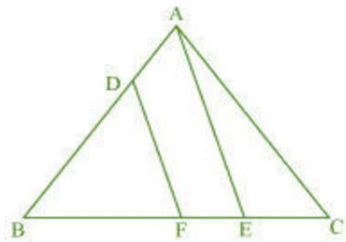
Given that, in ΔABC, DE || AC, therefore

According to Thales theorem, we have

$$\frac{BD}{DA} = \frac{BE}{EC} \quad \ldots (1)$$

Similarly,

In ΔABC, DF || AE, therefore



According to Thales theorem, we have

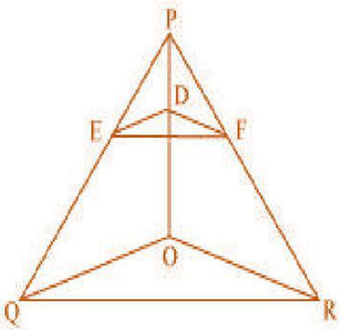
$$\frac{BD}{DA} = \frac{BF}{FE} \quad \ldots (2)$$

From the equation (1) and (2), we have

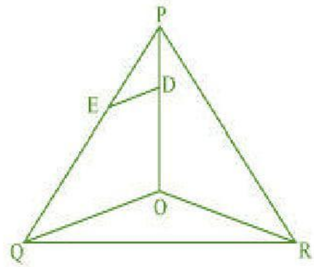
$$\frac{BF}{FE} = \frac{BE}{EC}$$

Question 5:

In Figure, DE || OQ and DF || OR. Show that EF || QR.



Answer 5:



Given that, in ΔPOQ, DE || OQ,

Therefore,

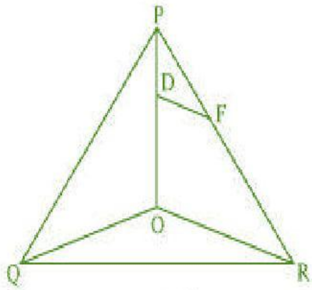
According to Thales theorem, we have

$$\frac{PE}{EQ} = \frac{PD}{DO} \quad \ldots (1)$$

Similarly,

In ΔPOR, DF || OR,

Therefore,



According to Thales theorem, we have

$$\frac{PF}{FR} = \frac{PD}{DO} \quad \ldots (2)$$

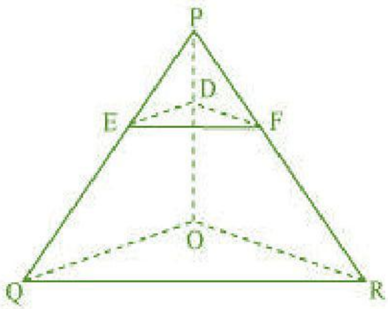
From the equation (1) and (2), we have

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

Now,

In triangle PQR,

$$\frac{PE}{EQ} = \frac{PF}{FR} \quad \text{[Proved above]}$$

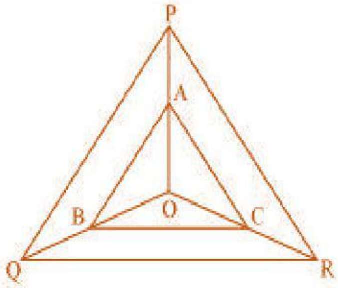


Therefore,

According to converse of Thales theorem, we have

EF || OR

Question 6:
In Figure, A, B and C are points on OP, OQ and OR respectively such that AB||PQ and AC||PR. Show that BC||QR.

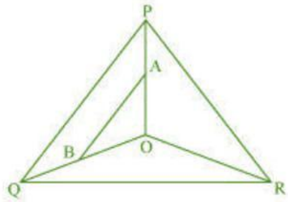


Answer 6:
Given that, in ΔPOQ, AB || PQ,

Therefore,

According to Thales theorem, we have

$\frac{OA}{AP} = \frac{OB}{BQ}$ ... (1)



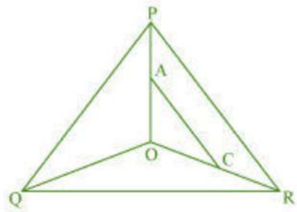
Similarly,

In ΔPOR, AC || PR,

Therefore,

According to Thales theorem, we have

$\frac{OA}{AP} = \frac{OC}{CR}$ ... (2)

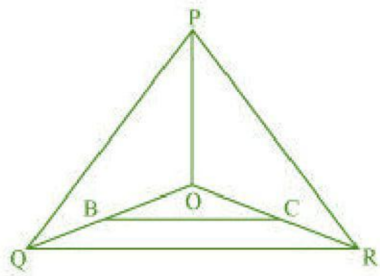


From the equation (1) and (2), we have

$\frac{OB}{OQ} = \frac{OC}{CR}$

Now, in triangle OQR,

$\frac{OB}{OQ} = \frac{OC}{CR}$ [Proved above]

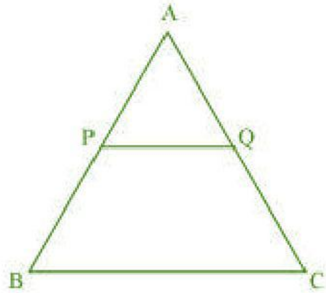


According to converse of Thales theorem, we have

BC || QR

Question 7:
Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer 7:
Let PQ is a line through the mid-point of AB, parallel to BC intersects AC at Q. i.e., PQ || BC,



In triangle ABC,

According to Thales theorem, we have

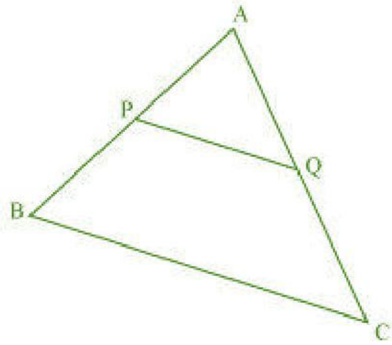
$\frac{AP}{PB} = \frac{AQ}{QC}$

$1 = \frac{AQ}{QC}$ [Because AP = PB]

$\Rightarrow AQ = QC$,

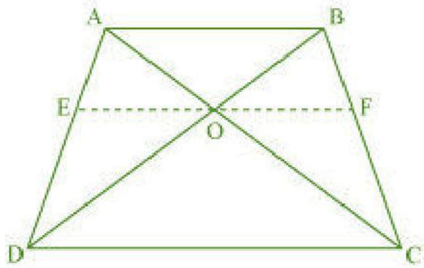
Hence, Q is the mid-point of AC.

Question 8:
Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer 8:


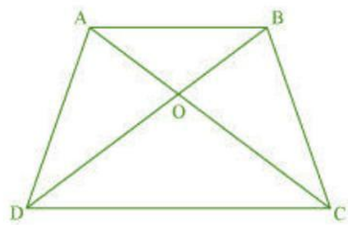
Let, PQ is a line, which passes through the mid-points of AB and AC.
Therefore, AP = PB and AQ = QC.
$\Rightarrow \frac{AP}{PB} = 1$ and $\frac{AQ}{QC} = 1$
or
$\frac{AP}{PB} = \frac{AQ}{QC} = 1$
Now, in triangle ABC,
$\frac{AP}{PB} = \frac{AQ}{QC}$ [Proved above]
Hence, according to converse of Thales theorem, we have, PQ || BC

Question 9:
ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$

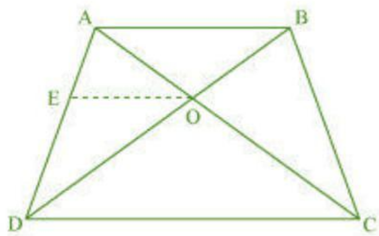
Answer 9:


A line is drawn through the point O, parallel to CD, such that EF||CD.
In ΔADC, EO || CD
According to Thales theorem, we have, $\frac{AE}{ED} = \frac{AO}{OC}$ ... (1)
Similarly, in ΔABD, EO || AB
According to Thales theorem, we have, $\frac{AE}{ED} = \frac{BO}{OD}$ ... (2)
From the equation (1) and (2), we get
$\frac{AO}{OC} = \frac{BO}{OD} \Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$

Question 10:
The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Answer 10:


A line is drawn through the point O, parallel to AB, such that EO||AB.
In ΔABD, EO || AB
According to Thales theorem, we have
$\frac{AE}{ED} = \frac{BO}{OD}$ ... (1)
But, given that
$\frac{AO}{BO} = \frac{CO}{DO}$
$\Rightarrow \frac{AO}{CO} = \frac{BO}{DO}$ ... (2)



From the equation (1) and (2), we have
$\frac{AE}{ED} = \frac{AO}{OC}$
⇒ EO || DC [According to converse of Thales theorem, we have]
⇒ AB || OE || DC
⇒ AB || CD
∴ ABCD is a trapezium.