Exercise 5.4 (Optional)*

Question 1:
Which term of the AP: 121, 117, 113 . . . is its first negative term?
[**Hint:** Find n for $a_n < 0$]

Answer 1:
Here, $a = 121$ and $d = 117 - 121 = -4$.
Let, a_n be the first negative term of this AP.
$\Rightarrow a_n < 0$
$\Rightarrow a + (n-1)d < 0$
$\Rightarrow 121 + (n-1)(-4) < 0$
$\Rightarrow 121 - 4n + 4 < 0$
$\Rightarrow 125 < 4n$
$\Rightarrow n > \frac{125}{4}$
$\Rightarrow n > 31.25$
$\Rightarrow n = 32$
Hence, 32nd term of the AP: 121, 117, 113 . . . is its first negative term.

Question 2:
The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Answer 2:
Let, the first term $= a$ and common difference $= d$
The sum of the third and the seventh terms of the AP is 6, therefore
$a_3 + a_7 = 6$
$\Rightarrow a + 2d + a + 6d = 6$
$\Rightarrow 2a + 8d = 6$
$\Rightarrow a + 4d = 3$
$\Rightarrow a = 3 - 4d$ ... (1)
The product of the third and the seventh terms of the AP is 8, therefore
$(a_3)(a_7) = 8$
$\Rightarrow (a + 2d)(a + 6d) = 8$
Putting the value of a from the equation (1), we get
$(3 - 2d)(3 + 2d) = 8$
$\Rightarrow 3^2 - 4d^2 = 8$
$\Rightarrow 4d^2 = 1 \quad \Rightarrow d = \pm\frac{1}{2}$
If, $d = \frac{1}{2}$,
Putting the value of d in the equation (1), we get
$a = 3 - 4\left(\frac{1}{2}\right) = 1$
The sum of the 16 terms of this AP is given by
$S_{16} = \frac{16}{2}[2a + (16-1)d] = 8\left[2(1) + 15\left(\frac{1}{2}\right)\right] = 76$
If, $d = -\frac{1}{2}$,
Putting the value of d in the equation (1), we get

$a = 3 - 4\left(-\frac{1}{2}\right) = 5$

The sum of the 16 terms of this AP is given by

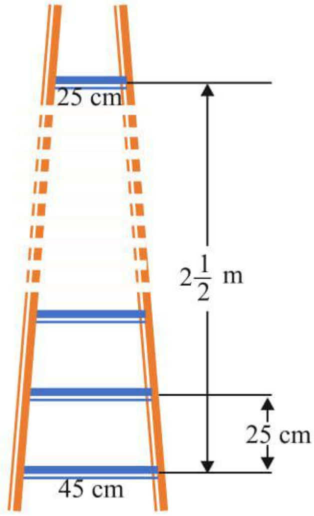
$S_{16} = \frac{16}{2}[2a + (16-1)d] = 8\left[2(5) + 15\left(-\frac{1}{2}\right)\right] = 20$

Hence, the sum of the 16 terms of this AP is 20 or 76.

Question 3:

A ladder has rungs 25 cm apart. (See Figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}\ m$ apart, what is the length of the wood required for the rungs?

[**Hint:** Number of rungs $= \frac{250}{25} + 1$]



Answer 3:

The distance between top and bottom rungs is $2\frac{1}{2}\ m$ and the distance between two successive rungs is 25 cm, therefore

The number of rungs $= \frac{250}{25} + 1 = 11$

The length of rungs is increasing from 25 cm to 45 cm in the form of AP, whose first term $a = 25$ and the last term $a_{11} = 45$ है।

Let the common difference of this AP be d.

Therefore, $a_{11} = 45$

$\Rightarrow a + (11-1)d = 45$

$\Rightarrow 25 + 10d = 45$

$\Rightarrow d = \frac{20}{10} = 2$

The length of wood required

$S_{11} = \frac{11}{2}[2a + (11-1)d]$

$= \frac{11}{2}[2(25) + 10(2)]$

$= 11 \times 35 = 385\ cm$

Hence, 385 cm length of the wood is required for the rungs.

Question 4:

The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

[**Hint:** $S_{x-1} = S_{49} - S_x$]

Answer 4:

Here, $a = 1$ and $d = 1$.

Sum to n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

The sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Therefore

$S_{x-1} = S_{49} - S_x$

$\Rightarrow \frac{x-1}{2}[2a + (x-1-1)d]$

$= \frac{49}{2}[2a + (49-1)d] - \frac{x}{2}[2a + (x-1)d]$

$\Rightarrow \frac{x-1}{2}[2(1) + (x-2)(1)]$

$= \frac{49}{2}[2(1) + 48(1)] - \frac{x}{2}[2(1) + (x-1)(1)]$

$\Rightarrow (x-1)[x] = 49[50] - x[x+1]$

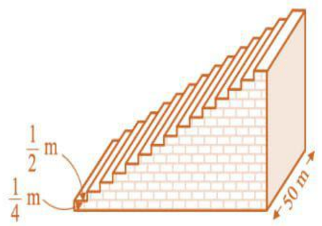
$\Rightarrow x^2 - x = 2450 - x^2 - x$

$\Rightarrow 2x^2 = 2450 \quad \Rightarrow x^2 = 1225 \quad \Rightarrow x = 35$

Hence, the value of x is 35.

Question 5:

A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}m$ and a tread of $\frac{1}{2}m$ (see Fig. 5.8). Calculate the total volume of concrete required to build the terrace.



[**Hint:** Volume of concrete required to build the first step $= \frac{1}{4} \times \frac{1}{2} \times 50\ m^3$]

Answer 5:

Volume of concrete required to build the first step $= \frac{1}{4} \times \frac{1}{2} \times 50 = \frac{1}{4} \times 25\ m^3$

Volume of concrete required to build the second step $= \frac{2}{4} \times \frac{1}{2} \times 50 = \frac{2}{4} \times 25\ m^3$

Volume of concrete required to build the third step $= \frac{3}{4} \times \frac{1}{2} \times 50 = \frac{3}{4} \times 25\ m^3$

Volume of steps are increasing in AP, whose first term $a = \frac{1}{4} \times 25$ and the last term $a_{15} = \frac{15}{4} \times 25$.

The common difference $d = \frac{2}{4} \times 25 - \frac{1}{4} \times 25 = \frac{1}{4} \times 25$.

Therefore, the total volume of concrete required to build the terrace = S_{15}

$= \frac{15}{2}\left[2\left(\frac{1}{4} \times 25\right) + (15-1)\left(\frac{1}{4} \times 25\right)\right]$

$= \frac{15}{2}\left[\frac{25}{2} + \frac{175}{2}\right]$

$= \frac{15}{2} \times \frac{200}{2}$

$= 750\ m^3$

Hence, the total $750\ m^3$ volume of concrete required to build the terrace.