Question 1:
A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm³.

Answer 1:
It can be observed that 1 round of wire will cover 3 mm height of cylinder.

\[
\text{Number of rounds} = \frac{\text{Height of cylinder}}{\text{Diameter of wire}} = \frac{12}{0.3} = 40 \text{ rounds}
\]

Length of wire required in 1 round = Circumference of base of cylinder
= \(2\pi r = 2\pi \times 5 = 10\pi\)

Length of wire in 40 rounds = \(40 \times 10\pi = \frac{400 \times 22}{7} = 8800\)

= 1257.14 cm = 12.57 m

Radius of wire = 0.15 cm

Volume of wire = Area of cross-section of wire \times Length of wire = \pi(0.15)^2 \times 1257.14 = 88.898 \text{ cm}^3

Mass = Volume \times \text{Density} = 88.898 \times 8.88 = 789.41 \text{ gm}

Question 2:
A right triangle whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of \(\pi\) as found appropriate.)

Answer 2:
The double cone so formed by revolving this right-angled triangle ABC about its hypotenuse is shown in the figure.

Hypotenuse \(AC = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ cm}\)

Area of \(\Delta ABC = \frac{1}{2} \times AB \times AC\)

\[
\frac{1}{2} \times AC \times OB = \frac{1}{2} \times 4 \times 3
\]

\[
\frac{1}{2} \times 5 \times OB = 6
\]

\[
OB = \frac{12}{5} = 2.4 \text{ cm}
\]

Volume of double cone = Volume of cone 1 + Volume of cone 2

\[
= \frac{1}{3} \pi r^2 h_1 + \frac{1}{3} \pi r^2 h_2
\]

\[
= \frac{1}{3} \pi r^2 (h_1 + h_2) = \frac{1}{3} \pi r^2 (OA + OC)
\]

\[
= \frac{1}{3} \times 3.14 \times (2.4)^2 \times 5
\]

= 30.14 cm³

Surface area of double cone = Surface area of cone 1 + Surface area of cone 2

= \(\pi rl_1 + \pi rl_2\)

= \(\pi r[4 + 3] = 3.14 \times 2.4 \times 7 = 52.75 \text{ cm}^2\)
Question 3:
A cistern, internally measuring 150 cm × 120 cm × 110 cm, has 129600 cm³ of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being 22.5 cm × 7.5 cm × 6.5 cm?

Answer 3:
Volume of cistern = 150 × 120 × 110 = 1980000 cm³
Volume to be filled in cistern = 1980000 – 129600 = 1850400 cm³
Let n numbers of porous bricks were placed in the cistern.
Volume of n bricks = n × 22.5 × 7.5 × 6.5 = 1096.875n
As each brick absorbs one-seventeenth of its volume, therefore, volume absorbed by these bricks

\[ \frac{n}{17} (1096.875) \]

\[ 1850400 + \frac{n}{17} (1096.875) = (1096.875)n \]

\[ 1850400 = \frac{16n}{17} (1096.875) \]

\[ n = 1792.41 \] Therefore, 1792 bricks were placed in the cistern.

Question 4:
In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is 97280 km², show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.

Answer 4:
Rainfall in the river valley = 10 cm = 0.1 m
Area of the river valley = 97280 km² = 97280000000 m²
Volume of the water = 97280000000×0.1 = 9728000000 m³
Now volume of one river water = 1072000×75×3 = 241200000 m³
Now volume of 3 rivers water = 3×241200000 = 843600000 m³
So, 9728000000 m³ ≈ 843600000 m³

Question 5:
An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see the given figure).

Answer 5:
Radius \( r_1 \) of upper circular end of frustum part 9 cm.
Radius \( r_2 \) of lower circular end of frustum part = Radius of circular end of cylindrical part = 4 cm
Height \( h_1 \) of frustum part = 22 – 10 = 12 cm, Height \( h_2 \) of cylindrical part = 10 cm
Slant height \( l \) of frustum part = \( \sqrt{(r_1 – r_2)^2 + h_2^2} = \sqrt{(9 – 4)^2 + 12^2} = 13 \) cm
Area of tin sheet required = CSA of frustum part + CSA of cylindrical part

\[ = \pi (r_1 + r_2)l + 2\pi r_2h_2 \]

\[ = \frac{22}{7} \times (9 + 4) \times 13 + 2 \times \frac{22}{7} \times 4 \times 10 \]

\[ = \frac{22}{7} \left[ 169 + 80 \right] = \frac{22 \times 249}{7} \]

\[ = 782 \frac{4}{7} \text{ cm}^2 \]
Question 6:
Derive the formula for the curved surface area and total surface area of the frustum of cone.

Answer 6:
Let ABC be a cone. A frustum DECB is cut by a plane parallel to its base. Let \( r_1 \) and \( r_2 \) be the radii of the ends of the frustum of the cone and \( h \) be the height of the frustum of the cone.

In \( \triangle AGB \) and \( \triangle ADF \), \( DF \parallel BG \)

\[ \triangle AGB \sim \triangle ADF \]

\[ \frac{DF}{AF} = \frac{AD}{AG} = \frac{AB}{BG} \]

\[ \frac{r_2}{h_1} = \frac{h_1 - h}{l_1} = \frac{l_1 - l}{l_1} \]

\[ \frac{r_2}{h_1} = \frac{1 - \frac{h}{h_1}}{1 - \frac{l}{l_1}} \]

\[ \frac{r_1}{l_1} = \frac{r_2}{l_1} \]

\[ \frac{r_1}{l_1} = \frac{r_2}{r_1} \]

\[ \frac{r_1}{l_1} = \frac{r_2}{r_1} \]

\[ \frac{l_1}{l_1} = \frac{r_1}{r_1} - \frac{r_2}{r_1} \]

\[ l_1 = \frac{r_2}{r_1} \]

CSA of frustum DECB = CSA of cone ABC – CSA cone ADE

\[ = \pi r_1 l_1 - \pi r_2 (l_1 - l) \]

\[ = \pi r_1 \left( \frac{l_1}{r_1 - r_2} \right) - \pi r_2 \left[ \frac{r_2 l}{r_1 - r_2} - l \right] \]

\[ = \pi r_1^2 - \pi r_2 \left[ \frac{r_1 l - r_2 l + r_2 l}{r_1 - r_2} \right] \]

\[ = \pi r_1^2 - \pi r_2 \left[ \frac{r_1 l - r_2 l}{r_1 - r_2} \right] \]

\[ = \frac{\pi l}{r_1 - r_2} \left[ r_1^2 - r_2^2 \right] \]

CSA of frustum = \( \pi (r_1 + r_2) l \)

Total surface area of frustum

\[ = \text{CSA of frustum} + \text{Area of upper circular end} + \text{Area of lower circular end} \]

\[ = \pi (r_1 + r_2) l + \pi r_1^2 + \pi r_2^2 = \pi [(r_1 + r_2) l + r_1^2 + r_2^2] \]

Question 7:
Derive the formula for the volume of the frustum of a cone.

Answer 7:
Let ABC be a cone. A frustum DECB is cut by a plane parallel to its base. Let \( r_1 \) and \( r_2 \) be the radii of the ends of the frustum of the cone and \( h \) be the height of the frustum of the cone.

In \( \triangle AGB \) and \( \triangle ADF \), \( DF \parallel BG \)

\[ \triangle AGB \sim \triangle ADF \]
Mathematics

(www.tiwariacademy.in)
(Chapter - 13) (Surface Areas and Volumes)
(Class 10)

\[
\frac{DF}{BG} = \frac{AF}{AG} = \frac{AD}{AB}
\]
\[
\frac{r_2}{r_1} = \frac{h_2 - h_1}{h_1} = \frac{l_2 - l_1}{l_1}
\]
\[
\frac{r_2}{r_1} = \frac{1 - \frac{h}{h_1}}{1 - \frac{l}{l_1}}
\]
\[
1 - \frac{h}{h_1} = \frac{r_2}{r_1}
\]
\[
\frac{h}{h_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1}
\]
\[
\frac{h_1}{h} = \frac{r_1}{r_2}
\]
\[
\frac{h}{h_1 - r_2} = \frac{r_1 h}{r_1 - r_2}
\]

Volume of frustum of cone = Volume of cone ABC – Volume of cone ADE

\[
= \frac{1}{3} \pi r_1^2 h_1 - \frac{1}{3} \pi r_2^2 (h_1 - h)
\]
\[
= \frac{\pi}{3} \left[ r_1^2 h_1 - r_2^2 (h_1 - h) \right]
\]
\[
= \frac{\pi}{3} \left[ r_1 \left( \frac{hr_1}{r_1 - r_2} \right) - r_2 \left( \frac{hr_2}{r_1 - r_2} - h \right) \right]
\]
\[
= \frac{\pi}{3} \left[ \left( \frac{hr_1^3}{r_1 - r_2} \right) - r_2 \left( \frac{hr_1 - hr_2 + hr_2}{r_1 - r_2} \right) \right]
\]
\[
= \frac{\pi}{3} \left[ \left( \frac{hr_1^3}{r_1 - r_2} \right) - \frac{hr_1 r_2}{r_1 - r_2} \right]
\]
\[
= \frac{\pi}{3} \left[ \frac{r_1 - r_2}{r_1 - r_2} \right]
\]
\[
= \frac{\pi}{3} \left[ \frac{(r_1 - r_2)(r_1^2 + r_2^2 + r_1 r_2)}{r_1 - r_2} \right]
\]
\[
= \frac{1}{3} \pi h \left[ r_1^2 + r_2^2 + r_1 r_2 \right]
\]