Mathematics

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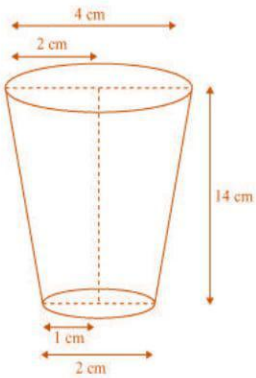
(Chapter – 13) (Surface Areas and Volumes)
(Class 10)

Exercise 13.4

Question 1:

A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Answer 1:



Radius of upper part of glass (r_1) = 4/2 = 2 cm
Radius of lower part of glass (r_2) = 2/2 = 1 cm
Height = 14 cm
Capacity of glass = Volume of frustum

$$= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2) = \frac{1}{3}\pi h(2^2 + 1^2 + 2 \times 1) = \frac{1}{3} \times \frac{22}{7} \times 14 \times (7)$$

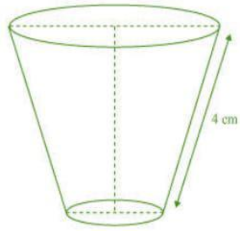
$$= \frac{1}{3} \times 22 \times 14 = \frac{308}{3} = 102\frac{2}{3}\ cm^3$$

Hence, the capacity of glass is $102\frac{2}{3}\ cm^3$.

Question 2:

The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Answer 2:



Circumference of upper part of frustum = 18 cm

$$\Rightarrow 2\pi r_1 = 18 \qquad \Rightarrow r_1 = \frac{9}{\pi}$$

Circumference of lower part of frustum = 6 cm

$$\Rightarrow 2\pi r_2 = 6 \qquad \Rightarrow r_2 = \frac{3}{\pi}$$

Height of frustum = 4 cm
Curved surface area of the frustum = $\pi\ (r_1 + r_2)\ l$

$$= \pi\left(\frac{9}{\pi} + \frac{3}{\pi}\right)4\ = 12 \times 4 = 48\ cm^2$$

Hence, the curved surface area of the frustum is 48 cm^2.

Question 3:

A *fez*, the cap used by the Turks, is shaped like the frustum of a cone (see Figure). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.

Answer 3:

Radius of lower part of cap (r_1) = 10 cm
Radius of upper part of cap (r_2) = 4 cm
Slant height of cap = 15 cm
Area of material used for making it = CSA of frustum + Area of upper part

$$= \pi(r_1 + r_2)l + \pi r_2$$

$$= \pi(10 + 4) \times 15 + \pi \times 4^2$$

$$= 210\pi + 16\pi = 226\pi$$

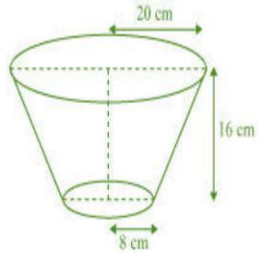
$$= 226 \times \frac{22}{7} = 710\frac{2}{7}\ cm^2$$

Hence, the area of material used for making is $710\frac{2}{7}\ cm^2$.

Question 4:

A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of ₹ 20 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹ 8 per 100 cm². (Take π = 3.14)

Answer 4:

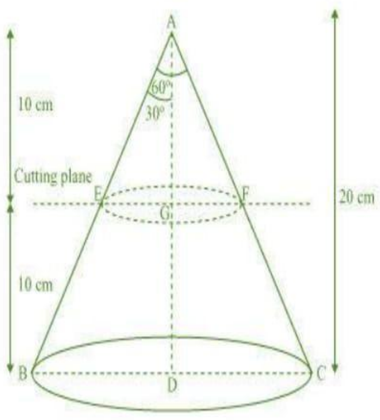


Radius of upper part of container (r_1) = 20 cm
Radius of lower part of container (r_2) = 8 cm
Height of container (h) = 16 cm
Slant height of container $= \sqrt{(r_1 - r_2)^2 + h^2}$
$= \sqrt{(20-8)^2 + 16^2} = \sqrt{144+256} = \sqrt{400} = 20\ cm$
Capacity of container = Volume of frustum $= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$
$= \frac{1}{3} \times 3.14 \times 16 \times (20^2 + 8^2 + 20 \times 8) = \frac{1}{3} \times 3.14 \times 16 \times (400 + 64 + 160)$
$= \frac{1}{3} \times 3.14 \times 16 \times 624 = 104449.92\ cm^3 = 10.45\ litres.$
Cost of 1 litre of milk = ₹ 20
Therefore, the cost of 10.45 litres of milk = 10.45 × ₹ 20 = ₹ 209
The area of metal sheet used to make the container
$= \pi\ (r_1 + r_2)\ l + \pi\ (r_2)^2$
$= \pi\ (20 + 8)\ 20 + \pi\ (8)^2$
$= 560\ \pi + 64\ \pi = 624\ \pi\ cm^2$
Cost of 100 cm² metal sheet = ₹ 8
Cost of 1 cm² metal sheet = ₹ $\frac{8}{100}$
Therefore, the cost of 624 π cm² metal sheet = ₹ $\frac{8}{100} \times 624\ \pi$ = ₹ $\frac{8}{100} \times 624 \times 3.14$ = ₹ 156.75
Hence, the cost of the milk which can completely fill the container is ₹ 209 and the cost of metal sheet used to make the container is ₹ 156.75.

Question 5:

A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16}$ cm find the length of the wire.

Answer 5:



In ΔAEG,
$\frac{EG}{AG} = \tan 30°$
$\Rightarrow \frac{EG}{10} = \frac{1}{\sqrt{3}}$
$\Rightarrow EG = \frac{10}{\sqrt{3}}$
In ΔABD,
$\frac{BD}{AD} = \tan 30°$
$\Rightarrow \frac{BD}{20} = \frac{1}{\sqrt{3}}$

$\Rightarrow BD = \frac{20}{\sqrt{3}}$

Radius of upper part of frustum (r_1) = $\frac{10}{\sqrt{3}}$

Radius of lower part of frustum (r_2) = $\frac{20}{\sqrt{3}}$

Height of frustum (h) = 10 cm

Volume of frustum $= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

$$= \frac{1}{3}\pi \times 10 \times \left[\left(\frac{10}{\sqrt{3}}\right)^2 + \left(\frac{20}{\sqrt{3}}\right)^2 + \frac{10}{\sqrt{3}} \times \frac{20}{\sqrt{3}}\right]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 10 \times \left(\frac{100}{3} + \frac{400}{3} + \frac{200}{3}\right)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 10 \times \left(\frac{700}{3}\right) = \frac{22000}{9}\ cm^3$$

Radius of wire (r) = $\frac{1}{2} \times \frac{1}{16} = \frac{1}{32}\ cm$

Let the length of wire = l

Volume of wire = area of cross-section of wire × length of wire

$= (\pi r^2)(l)$

$$= \pi\left(\frac{1}{32}\right)^2 \times l$$

Volume of frustum = volume of wire

$$\Rightarrow \frac{22000}{9} = \pi\left(\frac{1}{32}\right)^2 \times l$$

$$\Rightarrow \frac{22000}{9} = \frac{22}{7} \times \frac{1}{1024} \times l$$

$$\Rightarrow l = \frac{22000}{9} \times \frac{7}{22} \times 1024$$

$= 796444.44\ cm$

$= 7964.44\ m$

Hence, the length of wire is 7964.44 m.