Exercise 13.3

Question 1:
A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Answer 1:
Radius of metallic sphere (r_1) = 4.2 cm, Radius of cylinder (r_2) = 4.2 cm
Let, the height of cylinder = h
According to question, volume of sphere = volume of cylinder

$$\Rightarrow \frac{4}{3}\pi r_1^3 = \pi r_2^2 h \qquad \Rightarrow \frac{4}{3}\pi(4.2)^3 = \pi(6)^2 h$$

$$\Rightarrow h = \frac{4}{3} \times \frac{4.2 \times 4.2 \times 4.2}{36} = 1.4 \times 1.4 \times 1.4 = 2.74\ cm$$

Hence, the height of cylinder is 2.74 cm.

Question 2:
Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Answer 2:
Radius of first sphere (r_1) = 6 cm, Radius of second sphere (r_2) = 8 cm
Radius of third sphere (r_3) = 10 cm,
Let the radius of the new sphere = r
According to question, volumes of three spheres = volume of new sphere

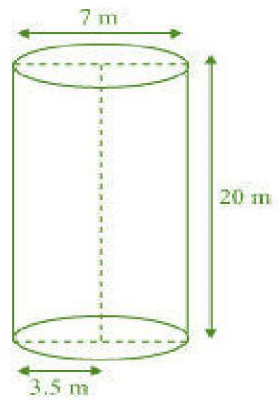
$$\Rightarrow \frac{4}{3}\pi(r_1^3 + r_2^3 + r_3^3) = \frac{4}{3}\pi r^3 \qquad \Rightarrow \frac{4}{3}\pi(6^3 + 8^3 + 10^3) = \frac{4}{3}\pi r^3$$

$$\Rightarrow 216 + 512 + 1000 = r^3 \qquad \Rightarrow r^3 = 1728 \qquad \Rightarrow r = 12\ cm$$

Hence, the radius of resulting sphere is 12 cm.

Question 3:
A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Answer 3:
Radius of well (r) = 7/2 m, Height of well = 20 m
Length of platform = 22 m, Width of platform = 14 m
Let the height of the platform = H
According to question, volume of the earth dug out = volume of earth of platform



$$\pi r^2 \text{h} = 22 \times 14 \times H$$

$$\Rightarrow \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 20 = 22 \times 14 \times H$$

$$\Rightarrow 11 \times 7 \times 10 = 22 \times 14 \times H$$

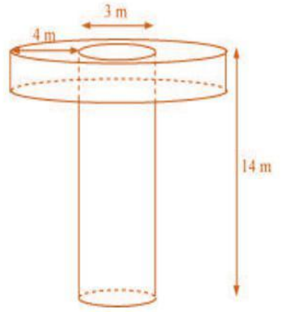
$$\Rightarrow \frac{11 \times 7 \times 10}{22 \times 14} = H \qquad \Rightarrow H = 2.5 \text{ m}$$

Hence, the height of the platform is 2.5 m.

Question 4:
A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Answer 4:

Radius of well (r) = $\frac{3}{2}$ m, Depth of well h_1 = 14 m, Width of embankment = 14 m



The embankment is in the form of hollow cylinder with internal radius $r_2 = \frac{3}{2}$ m

and Outer radius $r_1 = \frac{3}{2} + 4 = \frac{11}{2}$ m

Let the height of embankment = H

According to question, the earth dug out from the well = volume of embankment

$$\Rightarrow \pi r_1^2 h_1 = \pi(r_1^2 - r_2^2)h$$

$$\Rightarrow \pi\left(\frac{3}{2}\right)^2 \times 14 = \pi\left[\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] \times h \quad \Rightarrow \frac{9}{4} \times 14 = \frac{112}{4}h \quad \Rightarrow h = \frac{9}{8} = 1.125\ m$$

Hence, the height of embankment is 1.125 m.

Question 5:

A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Answer 5:

Radius of cylinder (r_1) = 12/2 = 6 cm, height of cylinder (h_1) = 15 cm
Height of conical part (h_2) = 12 cm, radius of conical part (r_2) = 6/2 = 3 cm
Radius of hemisphere (r_2) = 3 cm, let the number of ice cream cones = n

Therefore, volume of cylinder = n × [volume of cone + volume of hemisphere]

$$\Rightarrow \pi r_1^2 h_1 = n\left(\frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3\right) \Rightarrow 6^2 \times 15 = n\left(\frac{1}{3} \times 9 \times 12 + \frac{2}{3} \times 3^3\right)$$

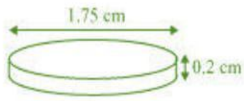
$$\Rightarrow 36 \times 15 = n(36 + 18) \qquad \Rightarrow n = \frac{36 \times 15}{54} = 10$$

Hence, the number of cones filled with ice cream is 10.

Question 6:

How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

Answer 6:



Radius of silver coin (r) = 1.75/2 = 0.875 cm, height of silver coin (h_1) = 0.2 cm

Length of cuboid = 5.5 cm, breadth of cuboid = 10 cm Height of cuboid = 3.5 cm,

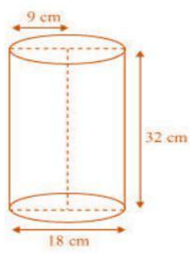
Let the number of coins = n

Therefore, n × volume of 1 silver coin = volume of cuboid ⇒ $n(\pi r^2 h_1) = lbh$

$$\Rightarrow n \times \frac{22}{7} \times (0.875)^2 \times 0.2 = 5.5 \times 10 \times 3.5 \Rightarrow n = \frac{5.5 \times 10 \times 3.5 \times 7}{0.875 \times 0.845 \times 0.2 \times 22} \Rightarrow n = 400$$

Hence, the number of coins is 400.

Question 7:



A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

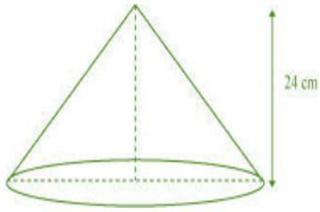
Answer 7:

Radius of bucket (r_1) = 18/2 = 9 cm, height of bucket (h_1) = 32 cm
Height of conical heap (h_2) = 24 cm,
Let the radius of conical heap = (r_2)

Therefore, volume of sand in the bucket = volume of conical heap

$\Rightarrow \pi r_1^2 h_1 = \frac{1}{3}\pi r_2^2 h_2$ $\qquad \Rightarrow \pi \times 18^2 \times 32 = \frac{1}{3}\pi \times r_2^2 \times 24$

$\Rightarrow r_2^2 = \frac{3 \times 18 \times 18 \times 32}{24} = 18 \times 18 \times 4$ $\qquad \Rightarrow r_2 = 36\ cm$



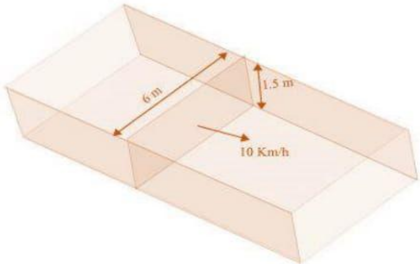
Slant height = $\sqrt{36^2 + 24^2} = \sqrt{1296 + 576} = \sqrt{1872} = 12\sqrt{13}\ cm$

Hence, the radius of conical heap is 36cm and its slant height is $12\sqrt{13}\ cm$.

Question 8:

Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Answer 8:



Width of canal = 6 m, depth of canal = 1.5 m

Speed of water = 10 km/h = $\frac{1000}{60}$ m/min

Volume of water in 1 minute = $6 \times 1.5 \times \frac{1000}{60}$ =1500 m^3

Therefore, the volume of water in 30 minutes

= 30 × 1500 = 45000 m^3

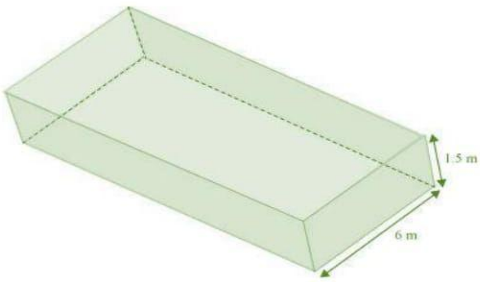
Let the area of irrigated field = A m^2

Therefore,

Volume of water from canal in 30 minutes = volume of water in the field

$\Rightarrow 45000 = \frac{A \times 8}{100}$

$\Rightarrow A = \frac{45000 \times 100}{8} = 562500\ m^2$



Hence, canal irrigate 562500 m^2 area in 30 minutes.

Question 9:

A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Answer 9:

Radius of pipe (r_1) = 20/200 = 0.1 m

Area of cross-section of pipe = $\pi {r_1}^2 = \pi(0.1)^2 = 0.01\pi\ m^3$

Speed of water = 3 km/h = 3000/60 = 50 m/min

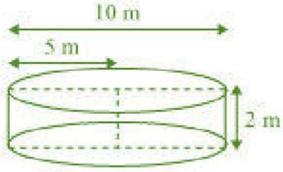
Volume of water flows through in 1 minute = 50 × 0.01π = 0.5π m^3

Volume of water flows through in t = t × 0.5π m^3

Radius of cylindrical tank (r_2) = 5 m

height of cylindrical tank (h_2) = 2 m

Let the tank will fill completely in t minutes.



Therefore,

The volume of water flows out in t minutes = volume of cylindrical tank

$\Rightarrow t \times 0.5\pi = \pi \times (r_2)^2 \times h_2$

$\Rightarrow t \times 0.5 = 5^2 \times 2$

$\Rightarrow t = 100$

Hence, the tank will completely fill in 100 minutes.