**Question 1:**
Find the area of the shaded region in Figure, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle. [Use $\pi = 22/7$]

**Answer 1:**
QR is diameter of circle. Therefore, $\angle RQP = 90^\circ$ [Angle in semicircle is right angle]

In $\triangle PQR$, by Pythagoras theorem,

$RP^2 + PQ^2 = RQ^2$

$(7)^2 + (24)^2 = RQ^2$

$\Rightarrow RQ^2 = 49 + 576 = 625$

$\Rightarrow RQ = \sqrt{625} = 25$

Therefore, the radius of circle = $RQ/2 = 25/2$ cm

Area of shaded region = Area of semicircle – Area of $\triangle PQR$

$= \frac{1}{2} \times \pi r^2 - \frac{1}{2} \times PR \times PQ = \frac{1}{2} \times \pi \left(\frac{25}{2}\right)^2 - \frac{1}{2} \times 7 \times 24$

$= \frac{625\pi}{4} - 84$

$= \frac{625 \times 22}{4 \times 7} - \frac{22 \times 25}{2 \times 2} - 7 \times 12$

$= \frac{6875 - 2352}{28} = \frac{4523}{28} \text{ cm}^2$

**Question 2:**
Find the area of the shaded region in Figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40^\circ$. [Use $\pi = 22/7$]

**Answer 2:**
Radius of smaller circle = 7 cm
Radius of larger circle = 14 cm

Area of shaded region

$= \frac{\angle AOC}{360^\circ} \times \pi (14)^2 - \frac{\angle AOC}{360^\circ} \times \pi (7)^2$

$= \frac{40^\circ}{360^\circ} \times \pi (14)^2 = \frac{40^\circ}{360^\circ} \times \pi (7)^2$

$= \frac{1}{9} \times 22 \times 14 \times 14 - \frac{1}{9} \times 22 \times 7 \times 7$

$= \frac{616 \times 154}{9} = \frac{154}{9} \text{ cm}^2$

**Question 3:**
Find the area of the shaded region in Figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles. [Use $\pi = 22/7$]

**Answer 3:**
Side of square is 14 cm, therefore, the radius of semicircle is 7 cm.

Area of semicircle

$= \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \pi (7)^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ cm}^2$

Area of square

$= (side)^2 = (14)^2 = 196 \text{ cm}^2$

Area of shaded region = Area of square – Area of two semicircles

$= 196 - 2 \times 77 = 196 - 154 = 42 \text{ cm}^2$
Question 4:
Find the area of the shaded region in Figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre. [Use $\pi = \frac{22}{7}$]

Answer 4:
We know that each angle of equilateral triangle is 60°.
Area of sector OCDE
$$\frac{60^\circ}{360^\circ} \times \pi r^2 = \frac{1}{6} \times \pi (6)^2 = \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7} \text{ cm}^2$$
Area of equilateral triangle OAB
$$\frac{\sqrt{3}}{4} (12)^2 = 36\sqrt{3} \text{ cm}^2$$
Area of circle
$$\pi r^2 = \pi (6)^2 = \frac{22}{7} \times 6 \times 6 = \frac{792}{7} \text{ cm}^2$$
Area of shaded region
$$= \text{Area of circle} - \text{Area of sector OCDE}$$
$$= \left(\frac{792}{7} + 36\sqrt{3} - \frac{132}{7}\right) \text{ cm}^2$$
$$= \left(36\sqrt{3} + \frac{660}{7}\right) \text{ cm}^2$$

Question 5:
From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in Figure. Find the area of the remaining portion of the square. [Use $\pi = \frac{22}{7}$]

Answer 5:
Radius of each quadrant = 1 cm
Area of each quadrant
$$\frac{90^\circ}{360^\circ} \times \pi r^2 = \frac{1}{4} \times \pi (1)^2 = \frac{1}{4} \times \frac{22}{7} \times 1 \times 1 = \frac{11}{14} \text{ cm}^2$$
Area of circle
$$\pi r^2 = \pi (1)^2 = \frac{22}{7} \times 1 \times 1 = \frac{22}{7} \text{ cm}^2$$
Area of square = (Side)$^2 = (4)^2 = 16 \text{ cm}^2$
Area of shaded region = Area of square - Area of circle - Area of 4 quadrants
$$16 - \frac{22}{7} - 4 \times \frac{11}{14} = 16 - \frac{22}{7} - \frac{22}{7} = \frac{112 - 44}{7} = \frac{68}{7} \text{ cm}^2$$

Question 6:
In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Figure. Find the area of the design (shaded region). [Use $\pi = \frac{22}{7}$]

Answer 6:
Radius of circle = 32
Centroid O divides median AD into 2:1, therefore AO:OD = 2:1.
$$\Rightarrow \frac{AO}{OD} = \frac{2}{1} \Rightarrow \frac{32}{OD} = \frac{2}{1} \Rightarrow OD = 16$$
Therefore, AD = 32 + 16 = 48 cm
In \(\triangle ABD\),

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\[ AB^2 = AD^2 + BD^2 = (48)^2 + \left(\frac{AB}{2}\right)^2 \]
\[ \Rightarrow \frac{3}{4} AB^2 = (48)^2 \Rightarrow AB = \frac{48 \times 2}{\sqrt{3}} = \frac{96\sqrt{3}}{3} = 32\sqrt{3} \]

Area of equilateral triangle ABC
\[ = \frac{\sqrt{3}}{4} (32\sqrt{3})^2 = 768\sqrt{3} \text{ cm}^2 \]

Area of circle
\[ = \pi r^2 = \pi (32)^2 = \frac{22}{7} \times 32 \times 32 = \frac{22528}{7} \text{ cm}^2 \]

Area of design = Area of circle – Area of equilateral triangle ABC
\[ = \left(\frac{22528}{7} - 768\sqrt{3}\right) \text{ cm}^2 \]

**Question 7:**
In Figure, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region. [Use \( \pi = \frac{22}{7} \)]

**Answer 7:**
The circles drawn taking A, B, C and D form quadrants of radius 7 cm in square.
Radius of each quadrant = 7 cm
Area of each quadrant
\[ = \frac{90^\circ}{360^\circ} \times \pi r^2 = \frac{1}{4} \times \pi (7)^2 = \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{2} \text{ cm}^2 \]

Area of square = (Side)^2 = (14)^2 = 196 cm^2
Area of shaded region = Area of square – Area of 4 quadrants
\[ = 196 - 4 \times \frac{77}{2} = 196 - 154 = 42 \text{ cm}^2 \]

**Question 8:**
Figure depicts a racing track whose left and right ends are semicircular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:
(i) the distance around the track along its inner edge
(ii) the area of the track.
[Use \( \pi = \frac{22}{7} \)]

**Answer 8:**
(i) The distance around the track along its inner edge
\[ = AB + \text{arc } BEC + CD + \text{arc } DFA \]
\[ = 106 + \frac{1}{2} \times 2\pi r + 106 + \frac{1}{2} \times 2\pi r \]
\[ = 106 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30 + 106 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 30 \]
\[ = 212 + 2 \times \frac{22}{7} \times 30 = 212 + \frac{1320}{7} = \frac{2804}{7} \text{ m} \]

(ii) Area of Track = (Area of GHJI – Area of ABCD) + (Area of semicircle HKI – Area of semicircle BEC) + (Area of semicircle GLJ – Area of semicircle AFD)
\[ = (106 \times 80 - 106 \times 60) + \frac{1}{2} \times \frac{22}{7} \times [(40)^2 - (30)^2] + \frac{1}{2} \times \frac{22}{7} \times [(40)^2 - (30)^2] \]
\[ = 106(80 - 60) + \frac{1}{2} \times \frac{22}{7} \times (700) + \frac{1}{2} \times \frac{22}{7} \times (700) = 2120 + \frac{22}{7} \times (700) = 2120 + 2200 = 4320 \]
Question 9:
In Figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region. [Use π = 22/7]

Answer 9:
Radius of smaller circle = 7/2 cm
Area of smaller circle = \( \pi r^2 = \pi \left( \frac{7}{2} \right)^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} \) cm²
Radius of larger circle = 7 cm
Area of semicircle AECFB
\( \frac{1}{2} \times \pi r^2 = \pi \left( \frac{7}{2} \right)^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \) cm²
Area of triangle ACB
\( \frac{1}{2} \times AB \times OC = \frac{1}{2} \times 14 \times 7 = 49 \) cm²
Area of shaded region
\( = \text{Area of smaller circle} + \text{Area of semicircle AECFB} - \text{Area of triangle ACB} \)
\( = \left( \frac{77}{2} + 77 - 49 \right) \) cm² = (38.5 + 28) cm² = 66.5 cm²

Question 10:
The area of an equilateral triangle ABC is 17320.5 cm². With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Figure). Find the area of the shaded region. [Use π = 3.14 and \( \sqrt{3} = 1.7325 \)]

Answer 10:
Let the each side of equilateral triangle = \( a \)
Area of equilateral triangle = 17320.5 cm²
\( \Rightarrow \frac{\sqrt{3}}{4} a^2 = 17320.5 \Rightarrow \frac{1.73205}{4} a^2 = 17320.5 \Rightarrow a^2 = 40000 \Rightarrow a = 200 \)
Area of sector ADEF
\( = \frac{\theta}{360°} \times \pi r^2 = \frac{60°}{360°} \times \frac{22}{7} (100)^2 = \frac{1}{6} \times 3.14 \times 100 \times 100 = \frac{15700}{3} \) cm²
Area of shaded region = Area of equilateral triangle – Area of three sectors
\( = 17320.5 \text{ cm}^2 - 3 \times \frac{15700}{3} \text{ cm}^2 \)
\( = 17320.5 \text{ cm}^2 - 15700 \text{ cm}^2 \)
\( = 1620.5 \text{ cm}^2 \)

Question 11:
On a square handkerchief, nine circular designs each of radius 7 cm are made (see Figure). Find the area of the remaining portion of the handkerchief. [Use π = 22/7]

Answer 11:
Radius of circle = 7 cm
Area of one circular design
\( = \pi r^2 = \pi (7)^2 = \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2 \)
Side of square = 42 cm
Area of square = (Side)² = (42)² = 1764 cm²
The area of the remaining portion = Area of square – Area of 9 circular designs
\( = 1764 - 9 \times 154 = 196 - 1386 = 378 \text{ cm}^2 \)
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Question 12:
In Figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the
(i) quadrant OACB,
(ii) shaded region. [Use π = 22/7]

Answer 12:
(i) Radius of quadrant = 1 cm
Area of quadrant
\[ \frac{90^\circ}{360^\circ} \times \pi r^2 = \frac{1}{4} \times \pi (3.5)^2 \]
\[ = \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 = \frac{77}{8} \text{ cm}^2 \]

Area of triangle OBD
\[ \frac{1}{2} \times OB \times OD = \frac{1}{2} \times 3.5 \times 2 = 3.5 \text{ cm}^2 \]

(ii) Area of shaded region = Area of quadrant – Area of triangle OBD
\[ \frac{77}{8} - 3.5 = \frac{77}{8} - \frac{7}{2} = \frac{77 - 28}{8} = \frac{49}{8} \text{ cm}^2 \]

Question 13:
In Figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region.
(Use π = 3.14) [Use π = 22/7]

Answer 13:
In ΔOAB,
\[ OB^2 = OA^2 + AB^2 \rightarrow OB^2 = (20)^2 + (20)^2 \rightarrow OB^2 = 400 + 400 \]
\[ \Rightarrow OB^2 = 800 \Rightarrow OB = \sqrt{800} \Rightarrow OB = 20\sqrt{2} \text{ cm} \]
Radius of quadrant = 20\sqrt{2} cm

Area of quadrant
\[ \frac{90^\circ}{360^\circ} \times \pi r^2 = \frac{1}{4} \times \pi (20\sqrt{2})^2 \]
\[ = \frac{1}{4} \times 3.14 \times 20\sqrt{2} \times 20\sqrt{2} = 628 \text{ cm}^2 \]

Area of square
\[ = (Side)^2 = (20)^2 = 400 \text{ cm}^2 \]
Area of shaded region = Area of quadrant – Area of square
\[ = 628 - 400 = 228 \text{ cm}^2 \]

Question 14:
AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see Figure). If \( \angle AOB = 30^\circ \), find the area of the shaded region. [Use π = 22/7]

Answer 14:
Area of shaded region = Area of sector OAEB – Area of sector OCFD
\[ \frac{30^\circ}{360^\circ} \times \pi \times (21)^2 - \frac{30^\circ}{360^\circ} \times \pi \times 7^2 \]
\[ = \frac{1}{12} \times \pi [441 - 49] \]
\[ = \frac{1}{12} \times \frac{22}{7} \times 392 = \frac{308}{3} \text{ cm}^2 \]
**Question 15:**

In Figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region. [Use $\pi = \frac{22}{7}$]

**Answer 15:**

Radius of sector = 14 cm

Area of sector

$$= \frac{90^\circ}{360^\circ} \times \pi r^2 = \frac{1}{4} \times \pi (14)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154 \text{ cm}^2$$

In ΔABC,

$$BC^2 = AC^2 + AB^2 \Rightarrow BC^2 = (14)^2 + (14)^2 \Rightarrow BC^2 = 196 + 196$$

$$\Rightarrow BC^2 = 392 \Rightarrow BC = \sqrt{392} \Rightarrow BC = 14\sqrt{2}$$

Therefore, the diameter of semicircle = $BC = 14\sqrt{2}$

Radius of semicircle = $7\sqrt{2} \text{ cm}$

Area of semicircle

$$= \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \pi (7\sqrt{2})^2 = \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2} = 154 \text{ cm}^2$$

Area of triangle ABC

$$= \frac{1}{2} \times AC \times AB = \frac{1}{2} \times 14 \times 14 = 98 \text{ cm}^2$$

Area of shaded region

= Area of triangle ABC + Area of semicircle - Area of quadrant

= $(98 + 154 - 154) \text{ cm}^2$

= $98 \text{ cm}^2$

**Question 16:**

Calculate the area of the designed region in Figure common between the two quadrants of circles of radius 8 cm each. [Use $\pi = \frac{22}{7}$]

**Answer 16:**

Area of sector DAFC

$$= \frac{90^\circ}{360^\circ} \times \pi r^2 = \frac{1}{4} \times \pi (8)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 8 \times 8 = \frac{352}{7} \text{ cm}^2$$

Area of triangle ADC

$$= \frac{1}{2} \times DC \times AD$$

$$= \frac{1}{2} \times 8 \times 8$$

= $32 \text{ cm}^2$

Area of segment = Area of sector DAFC - Area of triangle ADC

$$= \left(\frac{352}{7} - 32\right) \text{ cm}^2 = \left(\frac{352 - 224}{7}\right) \text{ cm}^2 = \left(\frac{128}{7}\right) \text{ cm}^2$$

Area of shaded region = Area of two segments

$$= 2 \times \left(\frac{128}{7}\right) \text{ cm}^2 = \frac{256}{7} \text{ cm}^2$$