Mathematics
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(Chapter - 1) (Real Numbers)
(Class X)
Exercise 1.4

**Question 1:**
Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) \(\frac{13}{3125}\)  
(ii) \(\frac{17}{8}\)  
(iii) \(\frac{64}{455}\)  
(iv) \(\frac{15}{1600}\)

(v) \(\frac{29}{343}\)  
(vi) \(\frac{23}{2^35^2}\)  
(vii) \(\frac{129}{2^35^77^5}\)  
(viii) \(\frac{5}{15}\)

**Answer 1:**

(i) \(\frac{13}{3125}\)

3125 = 5×5×5×5×5 = 5\(^5\)

The denominator is of the form 5\(^m\).

Hence, the decimal expansion of \(\frac{13}{3125}\) is terminating.

(ii) \(\frac{17}{8}\)

8 = 2×2×2 = 2\(^3\)

The denominator is of the form 2\(^m\).

Hence, the decimal expansion of \(\frac{17}{8}\) is terminating.

(iii) \(\frac{64}{455}\)

455 = 4×7×13

Since the denominator is not in the form 2\(^m\)×5\(^n\), and it also contains 7 and 13 as its factors, its decimal expansion will be non-terminating repeating.

(iv) \(\frac{15}{1600}\)

1600 = 2×2×2×2×2×2×5×5 = 2\(^6\)5\(^2\)

The denominator is of the form 2\(^m\)×5\(^n\).

Hence, the decimal expansion of \(\frac{15}{1600}\) is terminating.

(v) \(\frac{29}{343}\)

343 = 7×7×7 = 7\(^3\)

Since the denominator is not in the form 2\(^m\)×5\(^n\), and it has 7 as its factor, the decimal expansion of \(\frac{29}{343}\) is non-terminating repeating.

(vi) \(\frac{23}{2^35^2}\)

Denominator = 2\(^3\)5\(^2\)

The denominator is of the form 2\(^m\)×5\(^n\).

Hence, the decimal expansion of \(\frac{23}{2^35^2}\) is terminating.

(vii) \(\frac{129}{2^35^77^5}\)

Since the denominator is not of the form 2\(^m\)×5\(^n\), and it also has 7 as its factor, the decimal expansion of \(\frac{129}{2^35^77^5}\) is non-terminating repeating.
(viii) \[ \frac{6}{15} = \frac{2 \times 3}{5 \times 3} = \frac{2}{5} \]

The denominator is of the form \(5^n\).

Hence, the decimal expansion of \(\frac{6}{15}\) is terminating.

(ix) \[ \frac{35}{50} = \frac{5 \times 7}{2 \times 5 \times 5} = \frac{7}{10} \]

Denominator = \(10 = 2 \times 5\)

The denominator is of the form \(2^m \times 5^n\).

Hence, the decimal expansion of \(\frac{35}{50}\) is terminating.

(x) \[ \frac{77}{210} = \frac{7 \times 11}{2 \times 3 \times 5 \times 7} = \frac{11}{30} \]

Denominator = \(30 = 2 \times 3 \times 5\)

Since the denominator is not of the form \(2^m \times 5^n\), and it also has 3 as its factors, the decimal expansion of \(\frac{77}{210}\) is non-terminating repeating.

**Question 2:**
Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

**Answer 2:**

(i) \[ \frac{13}{3125} = \frac{13}{5 \times 5 \times 5 \times 5 \times 5} = \frac{13 \times 5^5}{25^5} = \frac{13 \times 32}{10^5} = \frac{416}{10^5} = 0.00416 \]

(ii) \[ \frac{17}{8} = \frac{17}{2 \times 2 \times 2} = \frac{17 \times 5^3}{2^3 \times 5^3} = \frac{17 \times 125}{(2 \times 5)^3} = \frac{2125}{10^3} = 2.125 \]

(iii) \[ \frac{64}{455} \]

Decimal expansion is non-terminating repeating.

(iv) \[ \frac{15}{1600} = \frac{3 \times 5}{2^6 \times 5} = \frac{3}{2^6 \times 5^3} = \frac{3 \times 3125}{(2 \times 5)^6} = \frac{9375}{10^6} = 0.009375 \]

(v) \[ \frac{29}{343} \]

Decimal expansion is non-terminating repeating.

(vi) \[ \frac{23}{2^5 \times 5^2} = \frac{23 \times 5}{2^5 \times 5^3} = \frac{115}{10^3} = 0.115 \]
(vii) \( \frac{129}{2^{5}7^{2}5} \)

Decimal expansion is non-terminating repeating.

(viii) \( \frac{6}{15} \)

\[
\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2 \times 2}{2 \times 5} = \frac{4}{10} = 0.4
\]

(ix) \( \frac{35}{50} \)

\[
\frac{35}{50} = \frac{5 \times 7}{2 \times 5 \times 5} = \frac{7}{2 \times 5} = \frac{7}{10} = 0.7
\]

(x) \( \frac{77}{210} \)

Decimal expansion is non-terminating repeating.

**Question 3:**

The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form \( \frac{p}{q} \), what can you say about the prime factor of \( q \)?

(i) 43.123456789  
(ii) 0.120120012000120000...  
(iii) 43.123456789

**Answer 3:**

(i) 43.123456789

Since this number has a terminating decimal expansion, it is a rational number of the form \( \frac{p}{q} \) and \( q \) is of the form \( 2^m \times 5^n \)

i.e., the prime factors of \( q \) will be either 2 or 5 or both.

(ii) 0.120120012000120000...

The decimal expansion is neither terminating nor recurring. Therefore, the given number is an irrational number.

(iii) 43.123456789

Since the decimal expansion is non-terminating recurring, the given number is a rational number of the form \( \frac{p}{q} \) and \( q \) is not of the form \( 2^m \times 5^n \) i.e., the prime factors of \( q \) will also have a factor other than 2 or 5.