Mathematics
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(Chapter - 1) (Real Numbers)
(Class X)
Exercise 1.3

Question 1:
Prove that $\sqrt{5}$ is irrational.

**Answer 1:**
Let $\sqrt{5}$ is a rational number.
Therefore, we can find two integers $a, b \ (b \neq 0)$ such that $\sqrt{5} = \frac{a}{b}$ Let $a$ and $b$ have a common factor other than 1. Then we can divide them by the common factor, and assume that $a$ and $b$ are co-prime.

$$a = \sqrt{5}b$$
$$\Rightarrow a^2 = 5b^2$$

Therefore, $a^2$ is divisible by 5 and it can be said that $a$ is divisible by 5.
Let $a = 5k$, where $k$ is an integer

$$(5k)^2 = 5b^2$$
$$\Rightarrow 25k^2 = b^2$$

This means that $b^2$ is divisible by 5 and hence, $b$ is divisible by 5.
This implies that $a$ and $b$ have 5 as a common factor.
And this is a contradiction to the fact that $a$ and $b$ are co-prime.
Hence, $\sqrt{5}$ cannot be expressed as $\frac{p}{q}$ or it can be said that $\sqrt{5}$ is irrational.

Question 2:
Prove that $3 + 2\sqrt{5}$ is irrational.

**Answer 2:**
Let $3 + 2\sqrt{5}$ is rational.
Therefore, we can find two co-prime integers $a, b \ (b \neq 0)$ such that

$$3 + 2\sqrt{5} = \frac{a}{b}$$
$$\Rightarrow 2\sqrt{5} = \frac{a}{b} - 3$$
$$\Rightarrow \sqrt{5} = \frac{1}{2} \left( \frac{a}{b} - 3 \right)$$

Since $a$ and $b$ are integers, $\frac{1}{2} \left( \frac{a}{b} - 3 \right)$ will also be rational and therefore, $\sqrt{5}$ is rational.
This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3 + 2\sqrt{5}$ is rational is false.
Therefore, $3 + 2\sqrt{5}$ is irrational.

Question 3:
Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$  
(ii) $7\sqrt{5}$  
(iii) $6 + \sqrt{2}$

**Answer 3:**
(i) $\frac{1}{\sqrt{2}}$
Let $\frac{1}{\sqrt{2}}$ is rational.
Therefore, we can find two co-prime integers $a, b \ (b \neq 0)$ such that

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$

Or
\[ \frac{b}{a} \text{ is rational as } a \text{ and } b \text{ are integers.} \]

Therefore, \( \sqrt{2} \) is rational which contradicts to the fact that \( \sqrt{2} \) is irrational.

Hence, our assumption is false and \( \frac{1}{\sqrt{2}} \) is irrational.

(ii) \( 7\sqrt{5} \)

Let \( 7\sqrt{5} \) is rational.

Therefore, we can find two co-prime integers \( a, b \) \((b \neq 0)\) such that

\[ 7\sqrt{5} = \frac{a}{b} \]

\[ \Rightarrow \sqrt{5} = \frac{a}{7b} \]

\( \frac{a}{7b} \) is rational as \( a \) and \( b \) are integers.

Therefore, \( \sqrt{5} \) should be rational.

This contradicts the fact that \( \sqrt{5} \) is irrational. Therefore, our assumption that \( 7\sqrt{5} \) is rational is false.

Hence, \( 7\sqrt{5} \) is irrational.

(iii) \( 6 + \sqrt{2} \)

Let \( 6 + \sqrt{2} \) be rational.

Therefore, we can find two co-prime integers \( a, b \) \((b \neq 0)\) such that

\[ 6 + \sqrt{2} = \frac{a}{b} \]

\[ \Rightarrow \sqrt{2} = \frac{a}{b} - 6 \]

Since \( a \) and \( b \) are integers, \( \frac{a}{b} - 6 \) is also rational and hence, \( \sqrt{2} \) should be rational. This contradicts the fact that \( \sqrt{2} \) is irrational. Therefore, our assumption is false and hence, \( 6 + \sqrt{2} \) is irrational.