Exercise 1.1

Question 1:

Use Euclid's division algorithm to find the HCF of:

(i) 135 and 225 (ii) 196 and 38220 (iii) 867 and 255

Answer 1:

(i) 135 and 225

Since 225 > 135, we apply the division lemma to 225 and 135 to obtain

$225 = 135 \times 1 + 90$

Since remainder $90 \neq 0$, we apply the division lemma to 135 and 90 to obtain

$135 = 90 \times 1 + 45$

We consider the new divisor 90 and new remainder 45, and apply the division lemma to obtain $90 = 2 \times 45 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 45, therefore, the HCF of 135 and 225 is 45.

(ii) 196 and 38220

Since 38220 > 196, we apply the division lemma to 38220 and 196 to obtain

$38220 = 196 \times 195 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 196, therefore, HCF of 196 and 38220 is 196.

(iii) 867 and 255

Since 867 > 255, we apply the division lemma to 867 and 255 to obtain

$867 = 255 \times 3 + 102$

Since remainder $102 \neq 0$, we apply the division lemma to 255 and 102 to obtain

$255 = 102 \times 2 + 51$

We consider the new divisor 102 and new remainder 51, and apply the division lemma to obtain

$102 = 51 \times 2 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 51, Therefore, HCF of 867 and 255 is 51.

Question 2:

Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

Answer 2:

Let a be any positive integer and $b = 6$.

Then, by Euclid's algorithm, $a = 6q + r$ for some integer $q \geq 0$ and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$ Also,

$6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where k_1 is a positive integer

$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1$, where k_2 is an integer

$6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1$, where k_3 is an integer

Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where k is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2.

Hence, these expressions of numbers are odd numbers and therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$, or $6q + 5$.

Question 3:

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer 3:
HCF (616, 32) will give the maximum number of columns in which they can march. We can use Euclid's algorithm to find the HCF.
$616 = 32 \times 19 + 8$
$32 = 8 \times 4 + 0$
The HCF (616, 32) is 8. Therefore, they can march in 8 columns each.

Question 4:
Use Euclid's division lemma to show that the square of any positive integer is either of form $3m$ or $3m + 1$ for some integer m.
[**Hint:** Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

Answer 4:
Let a be any positive integer and $b = 3$. Using Euclid's Division Lemma, $a = 3q + r$ for some integer $q \geq 0$ where $r = 0, 1, 2$ because $0 \leq r < 3$.

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$

$$a^2 = (3q)^2 \ or \ (3q+1)^2 \ or \ (3q+2)^2$$
$$= (3q)^2 \ or \ 9q^2 + 6q + 1 \ or \ 9q^2 + 12q + 4$$
$$= 3 \times (3q^2) \ or \ 3 \times (3q^2 + 2q) + 1 \ or \ 3 \times (3q^2 + 4q + 1) + 1$$
$$= 3k_1 \ or \ 3k_2 + 1 \ or \ 3k_3 + 1$$

Where k_1, k_2 and k_3 are some positive integers.
Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

Question 5:
Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

Answer 5:
Let a be any positive integer and $b = 3$, using Euclid's Division Lemma, $a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$. Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$.
Therefore, every number can be represented as these three forms.
There are three cases.
Case 1: When $a = 3q$,
$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$
Where m is an integer such that $m = 3q^3$
Case 2: When $a = 3q + 1$,
$a^3 = (3q + 1)^3$
$a^3 = 27q^3 + 27q^2 + 9q + 1$
$a^3 = 9(3q^3 + 3q^2 + q) + 1 = 9m + 1$
Where m is an integer such that $m = (3q^3 + 3q^2 + q)$
Case 3: When $a = 3q + 2$,
$a^3 = (3q + 2)^3$
$a^3 = 27q^3 + 54q^2 + 36q + 8$
$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$
$a^3 = 9m + 8$
Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$
Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.